

# Solusi Pengayaan Matematika

## Edisi 8

### Nomor Soal: 71-80

71. Jika  $f(x) = \frac{a^x}{a^x + \sqrt{a}}$ , maka nilai  $f\left(\frac{1}{11}\right) + f\left(\frac{2}{11}\right) + f\left(\frac{3}{11}\right) + \dots + f\left(\frac{10}{11}\right)$

adalah ....

- A. 10                      B. 8                      C. 7                      D. 5                      E. 3

**Solusi: [D]**

$$f\left(\frac{1}{11}\right) + f\left(\frac{10}{11}\right) = \frac{a^{\frac{1}{11}}}{a^{\frac{1}{11}} + \sqrt{a}} + \frac{a^{\frac{10}{11}}}{a^{\frac{10}{11}} + \sqrt{a}} = \frac{a + a^{\frac{1}{11}}\sqrt{a} + a + a^{\frac{10}{11}}\sqrt{a}}{a + a^{\frac{1}{11}}\sqrt{a} + a^{\frac{10}{11}}\sqrt{a} + a} = 1$$

$$f\left(\frac{2}{11}\right) + f\left(\frac{9}{11}\right) = \frac{a^{\frac{2}{11}}}{a^{\frac{2}{11}} + \sqrt{a}} + \frac{a^{\frac{9}{11}}}{a^{\frac{9}{11}} + \sqrt{a}} = \frac{a + a^{\frac{2}{11}}\sqrt{a} + a + a^{\frac{9}{11}}\sqrt{a}}{a + a^{\frac{2}{11}}\sqrt{a} + a^{\frac{9}{11}}\sqrt{a} + a} = 1$$

$$f\left(\frac{3}{11}\right) + f\left(\frac{8}{11}\right) = \frac{a^{\frac{3}{11}}}{a^{\frac{3}{11}} + \sqrt{a}} + \frac{a^{\frac{8}{11}}}{a^{\frac{8}{11}} + \sqrt{a}} = \frac{a + a^{\frac{3}{11}}\sqrt{a} + a + a^{\frac{8}{11}}\sqrt{a}}{a + a^{\frac{3}{11}}\sqrt{a} + a^{\frac{8}{11}}\sqrt{a} + a} = 1$$

$$f\left(\frac{4}{11}\right) + f\left(\frac{7}{11}\right) = \frac{a^{\frac{4}{11}}}{a^{\frac{4}{11}} + \sqrt{a}} + \frac{a^{\frac{7}{11}}}{a^{\frac{7}{11}} + \sqrt{a}} = \frac{a + a^{\frac{4}{11}}\sqrt{a} + a + a^{\frac{7}{11}}\sqrt{a}}{a + a^{\frac{4}{11}}\sqrt{a} + a^{\frac{7}{11}}\sqrt{a} + a} = 1$$

$$f\left(\frac{5}{11}\right) + f\left(\frac{6}{11}\right) = \frac{a^{\frac{5}{11}}}{a^{\frac{5}{11}} + \sqrt{a}} + \frac{a^{\frac{6}{11}}}{a^{\frac{6}{11}} + \sqrt{a}} = \frac{a + a^{\frac{5}{11}}\sqrt{a} + a + a^{\frac{6}{11}}\sqrt{a}}{a + a^{\frac{5}{11}}\sqrt{a} + a^{\frac{6}{11}}\sqrt{a} + a} = 1$$

$$f\left(\frac{1}{11}\right) + f\left(\frac{2}{11}\right) + f\left(\frac{3}{11}\right) + \dots + f\left(\frac{10}{11}\right) = 1 + 1 + 1 + 1 + 1 = 5$$

72. Jika  $f(x) = \frac{9^x}{9^x + 3}$ , maka  $f\left(\frac{1}{2006}\right) + f\left(\frac{2}{2006}\right) + \dots + f\left(\frac{2005}{2006}\right)$  adalah ....

- A. 2005    B. 1005,0    C. 1002,5    D. 1002,0    E. 1000,5

**Solusi: [C]**

$$f(x) = \frac{9^x}{9^x + 3}$$

$$f(1-x) = \frac{9^{1-x}}{9^{1-x} + 3} = \frac{9}{9 + 3 \times 9^x} = \frac{3}{3 + 9^x}$$

$$f(x) + f(1-x) = \frac{9^x}{9^x+3} + \frac{3}{3+9^x} = \frac{9^x+3}{9^x+3} = 1$$

$$\text{Jadi, } \sum_{k=1}^{2005} f\left(\frac{k}{2006}\right) = \frac{2005}{2} = 1002,5$$

73. Jika  $f(x) = \frac{x-1}{x+1}$ , maka  $f\left(\frac{1}{x}\right) = \dots$

- A.  $-\frac{1}{f(x)}$     B.  $\frac{1}{f(x)}$     C.  $-f(x)$     D.  $\frac{1}{f(-x)}$     E.  $-\frac{1}{f(-x)}$

**Solusi: [C]**

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}-1}{\frac{1}{x}+1} = \frac{\frac{1-x}{x}}{\frac{1+x}{x}} = \frac{1-x}{1+x} = -\frac{x-1}{x+1} = -f(x)$$

74. Jika  $f(x) = \frac{x-1}{x+1}$ , maka  $f\left(-\frac{1}{x}\right) = \dots$

- A.  $-\frac{1}{f(x)}$     B.  $\frac{1}{f(x)}$     C.  $-f(x)$     D.  $\frac{1}{f(-x)}$     E.  $-\frac{1}{f(-x)}$

**Solusi: [A]**

$$f\left(-\frac{1}{x}\right) = \frac{-\frac{1}{x}-1}{-\frac{1}{x}+1} = \frac{-\frac{1+x}{x}}{\frac{-1+x}{x}} = \frac{-1-x}{-1+x} = -\frac{1+x}{1-x} = -\frac{1}{\frac{1-x}{1+x}} = -\frac{1}{f(x)}$$

75. Sebuah fungsi  $f$  didefinisikan untuk bilangan-bilangan tidak negatif  $n$  dan  $k$  sebagai berikut.

$$f(0, n) = n + 1$$

$$f(k, 0) = f(k-1, 1)$$

$$f(k+1, n+1) = f(k, f(k+1, n)).$$

Tentukanlah  $f(2, 2005)$ .

- A. 4.023    B. 4.013    C. 3.025    D. 2.025    E. 2.015

**Solusi: [B]**

$$f(1, 0) = f(0, 1) = 1 + 1 = 2$$

$$f(1, n+1) = f(0, f(1, n)) = f(1, n) + 1$$

$$f(1, m) = m + 2$$

$$f(2, 0) = f(1, 1) = 1 + 2 = 3$$

$$f(2, n+1) = f(1, f(2, n)) = f(2, n) + 2$$

$$f(2, m) = 2m + 3$$

$$f(2, 2005) = 2 \cdot 2005 + 3 = 4013$$

76. Diberikan fungsi  $f(x) = (x^2 + 4x - 2000)^{2005} + 2004$ . Jumlah angka-angka dari  $f(\sqrt{2005} - 2)$  adalah ....

A. 12                      B. 7                      C. 6                      D. 4                      E. 3

**Solusi: [B]**

$$x = \sqrt{2005} - 2$$

$$x + 2 = \sqrt{2005}$$

$$x^2 + 4x + 4 = 2005$$

$$x^2 + 4x = 2001$$

$$f(x) = (x^2 + 4x - 2000)^{2002} + 2004 = (2001 - 2000)^{2010} + 2004$$

$$= 1^{2010} + 2004 = 1 + 2004 = 2005$$

Jadi, jumlah angka-angka dari  $f = 2 + 0 + 0 + 5 = 7$ .

77. Jika  $x$  adalah bilangan real sehingga  $x^3 + 4x - 8 = 0$ , maka nilai  $\sqrt[10]{x^7 + 64x^2 + 896}$  adalah ....

A. 2                      B. 4                      C. 6                      E. 8                      E. 32

**Solusi: [A]**

$$\begin{aligned} \sqrt[10]{x^7 + 64x^2 + 896} &= \sqrt[10]{x \cdot (x^3)^2 + 64x^2 + 896} = \sqrt[10]{x \cdot (8 - 4x)^2 + 64x^2 + 896} \\ &= \sqrt[10]{64x - 64x^2 + 16x^3 + 64x^2 + 896} \\ &= \sqrt[10]{64x + 16x^3 + 896} = \sqrt[10]{16(x^3 + 4x) + 896} \\ &= \sqrt[10]{16(8) + 896} = \sqrt[10]{1024} = 2 \end{aligned}$$

78. Jika  $f$  adalah fungsi yang memenuhi  $f(xy) = \frac{f(x)}{y}$  untuk setiap bilangan real positif  $x$  dan  $y$ . Jika  $f(30) = 2005$ , maka nilai  $f(2005)$  adalah ....

A. 45                      B. 30                      C. 25                      D. 15                      E. 5

**Solusi: [B]**

$$f(xy) = \frac{f(x)}{y}$$

$$f(y) = \frac{f(1)}{y} \text{ untuk semua nilai } y$$

$$f(30) = \frac{f(1)}{30} = 2005$$

$$f(1) = 30 \times 2005$$

$$f(2005) = \frac{f(1)}{2005} = \frac{30 \times 2005}{2005} = 30$$

79. Suatu fungsi didefinisikan untuk semua bilangan bulat positif dan  $f(1) = 2006$  dan  $f(1) + f(2) + f(3) + \dots + f(n) = n^2 f(n)$  untuk semua  $n \geq 1$ . Tentukan nilai dari  $f(2005)$ .

- A.  $\frac{2}{1005}$       B.  $\frac{1}{2011}$       C.  $\frac{2}{2011}$       D.  $\frac{1}{2005}$       E.  $\frac{2}{1005}$

**Solusi: [E]**

$$f(1) + f(2) = 2^2 f(2)$$

$$3f(2) = f(1)$$

$$f(2) = \frac{1}{3} f(1) = \frac{2}{2 \cdot 3} f(1)$$

$$f(3) = \frac{2}{3 \cdot 4} f(1)$$

$$f(4) = \frac{2}{4 \cdot 5} f(1)$$

$$f(5) = \frac{2}{5 \cdot 6} f(1)$$

dan seterusnya

$$\text{Secara umum } f(n) = \frac{2}{n(n+1)} f(1)$$

$$f(2005) = \frac{2}{2005 \cdot 2006} \cdot 2006 = \frac{2}{2005}$$

80. Didefinisikan fungsi  $f$  dengan  $f(x) = \frac{4x + \sqrt{4x^2 - 1}}{\sqrt{2x+1} + \sqrt{2x-1}}$  untuk  $x \geq 1$ . Tentukan

jumlah  $f(1) + f(2) + f(3) + \dots + f(40)$

- A. 216      B. 361      C. 364      D. 366      E. 464

**Solusi: [C]**

Misalnya  $x = \sqrt{2n+1}$  dan  $y = \sqrt{2n-1}$ , sehingga

$$x^2 + y^2 = (\sqrt{2n+1})^2 + (\sqrt{2n-1})^2 = 2n+1 + 2n-1 = 4n$$

$$xy = \sqrt{2n+1} \times \sqrt{2n-1} = \sqrt{4n^2 - 1}$$

$$x^2 - y^2 = (\sqrt{2n+1})^2 - (\sqrt{2n-1})^2 = 2n+1 - 2n-1 = 2$$

$$\begin{aligned}
f(n) &= \frac{4n + \sqrt{4n^2 - 1}}{\sqrt{2n+1} + \sqrt{2n-1}} = \frac{x^2 + y^2 + xy}{x + y} = \frac{x^3 - y^3}{x^2 - y^2} = \frac{(\sqrt{2n+1})^3 - (\sqrt{2n-1})^3}{(\sqrt{2n+1})^2 - (\sqrt{2n-1})^2} \\
&= \frac{(2n+1)^{\frac{3}{2}} - (2n-1)^{\frac{3}{2}}}{2n+1 - 2n+1} = \frac{1}{2} \left[ (2n+1)^{\frac{3}{2}} - (2n-1)^{\frac{3}{2}} \right] \\
f(1) + f(2) + f(3) + \dots + f(40) &= \frac{1}{2} \left( 3^{\frac{3}{2}} - 1^{\frac{3}{2}} + 5^{\frac{3}{2}} - 3^{\frac{3}{2}} + 7^{\frac{3}{2}} - 5^{\frac{3}{2}} + \dots + 81^{\frac{3}{2}} - 79^{\frac{3}{2}} \right) \\
&= \frac{1}{2} \left( 81^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) = \frac{1}{2} (9^3 - 1) = \frac{1}{2} (729 - 1) \\
&= \frac{1}{2} (728) = 364
\end{aligned}$$