

Solusi Pengayaan Matematika

Edisi 6

Nomor Soal: 51-60

51. Berapakah nilai dari $({}^2\log 3)({}^3\log 4)({}^4\log 5)\dots({}^{2047}\log 2048)$?

- A. 8 B. 9 C. 10 D. 11 E. 12

Solusi: [D]

Konsep: ${}^a\log b \times {}^b\log c = {}^a\log c$

$$({}^2\log 3)({}^3\log 4)({}^4\log 5)\dots({}^{2047}\log 2048) = {}^2\log 2048 = 11$$

52. Nilai dari $\frac{1}{{}^2\log a} + \frac{1}{{}^3\log a} + \frac{1}{{}^4\log a} + \dots + \frac{1}{{}^{100}\log a}$

- A. $a^{100!}$ B. ${}^a\log 1000!$ C. ${}^a\log 100!$ D. $100!$ E. ${}^{100!}\log a$

Solusi: [C]

$$\begin{aligned} \frac{1}{{}^2\log a} + \frac{1}{{}^3\log a} + \frac{1}{{}^4\log a} + \dots + \frac{1}{{}^{100}\log a} &= {}^a\log 2 + {}^a\log 3 + {}^a\log 4 + \dots + {}^a\log 100 \\ &= {}^a\log(2 \times 3 \times 4 \times \dots \times 100) = {}^a\log 100! \end{aligned}$$

53. Jika $a \cdot b \cdot c = 900$ dan ${}^2\log a = {}^3\log b = {}^5\log c$, maka nilai dari $a + b + c$ adalah

- A. 4 B. 9 C. 25 D. 34 E. 48

Solusi: [E]

$${}^2\log a = {}^3\log b = {}^5\log c = k$$

$${}^2\log a = k \Leftrightarrow 2^k = a$$

$${}^3\log b = k \Leftrightarrow 3^k = b$$

$${}^5\log c = k \Leftrightarrow 5^k = c$$

$$a \cdot b \cdot c = 900$$

$$2^k \cdot 3^k \cdot 5^k = 900$$

$$(2 \cdot 3 \cdot 5)^k = 900$$

$$30^k = 30^2$$

$$k = 2$$

$$k = 2 \rightarrow a = 2^k = 2^2 = 4$$

$$k = 2 \rightarrow b = 3^k = 3^2 = 9$$

$$k = 2 \rightarrow c = 5^k = 5^2 = 25$$

$$\text{Jadi, } a + b + c = 4 + 9 + 25 = 38.$$

54. Diberikan a , b , dan c lebih dari 1 dan d bilangan positif sehingga ${}^a \log d = 48$,
 ${}^b \log d = 80$, dan ${}^{abc} \log d = 48$. Tentukan ${}^c \log d$.

Solusi 1:

$${}^a \log d = 48 \Leftrightarrow a^{48} = d \Leftrightarrow a^{240} = d^5$$

$${}^b \log d = 80 \Leftrightarrow b^{80} = d \Leftrightarrow b^{240} = d^3$$

$${}^{abc} \log d = 24 \Leftrightarrow (abc)^{24} = d \Leftrightarrow (abc)^{240} = d^{10}$$

$$a^{240} b^{240} c^{240} = d^{10}$$

$$d^5 d^3 c^{240} = d^{10}$$

$$c^{240} = d^2$$

$$c^{120} = d$$

$${}^c \log d = 120$$

Solusi 2:

$${}^a \log d = 48 \Leftrightarrow \frac{\log d}{\log a} = 48$$

$${}^b \log d = 80 \Leftrightarrow \frac{\log d}{\log b} = 80$$

$${}^{abc} \log d = 24 \Leftrightarrow \frac{\log d}{\log abc} = 24$$

$$\frac{\log abc}{\log d} = \frac{1}{24}$$

$$\frac{\log a + \log b + \log c}{\log d} = \frac{1}{24}$$

$$\frac{\log a}{\log d} + \frac{\log b}{\log d} + \frac{\log c}{\log d} = \frac{1}{24}$$

$$\frac{1}{48} + \frac{1}{80} + \frac{\log c}{\log d} = \frac{1}{24}$$

$$\frac{\log c}{\log d} = \frac{1}{24} - \frac{1}{48} - \frac{1}{80}$$

$$\frac{\log c}{\log d} = \frac{10 - 5 - 3}{120} = \frac{2}{240} = \frac{1}{120}$$

$$\frac{\log d}{\log c} = 120$$

$${}^c \log d = 120$$

55. Tentukan nilai m jika ${}^8 \log m + {}^4 \log n^2 = 10$ dan ${}^8 \log n + {}^4 \log m^2 = 14$.

Solusi 1:

$${}^8 \log m + {}^4 \log n^2 = 10$$

$$\frac{\log m}{\log 8} + \frac{\log n^2}{\log 4} = 10$$

$$\frac{\log m}{3 \log 2} + \frac{2 \log n}{2 \log 2} = 10$$

$$\frac{\log m + 3 \log n}{3 \log 2} = 10$$

$$\frac{\log mn^3}{3 \log 2} = 10$$

$$\log mn^3 = 30 \log 2$$

$$\log mn^3 = \log 2^{30}$$

$$mn^3 = 2^{30} \dots (1)$$

$${}^8 \log n + {}^4 \log m^2 = 14$$

$$\frac{\log n}{\log 8} + \frac{\log m^2}{\log 4} = 14$$

$$\frac{\log n}{3 \log 2} + \frac{\log m}{\log 2} = 14$$

$$\frac{\log n + 3 \log m}{3 \log 2} = 14$$

$$\frac{\log nm^3}{3 \log 2} = 14$$

$$\log nm^3 = 42 \log 2$$

$$\log nm^3 = \log 2^{42}$$

$$nm^3 = 2^{42} \dots (2)$$

Persamaan (1) \times Persamaan (2) menghasilkan

$$m^4 n^4 = 2^{72}$$

$$mn = 2^{18}$$

Solusi 2:

$${}^8 \log m + {}^4 \log n^2 = 10$$

$$\frac{\log m}{\log 8} + \frac{\log n^2}{\log 4} = 10$$

$$\frac{\log m}{\log 8} + \frac{2 \log n}{\log 4} = 10 \dots (1)$$

$${}^8 \log n + {}^4 \log m^2 = 14$$

$$\frac{\log n}{\log 8} + \frac{\log m^2}{\log 4} = 14$$

$$\frac{\log n}{\log 8} + \frac{2\log m}{\log 4} = 14 \dots (2)$$

Hasil penjumlahan persamaan (1) dan (2) adalah

$$\frac{\log m}{\log 8} + \frac{2\log n}{\log 4} + \frac{\log n}{\log 8} + \frac{2\log m}{\log 4} = 10 + 14$$

$$\frac{\log m + \log n}{\log 8} + \frac{2(\log m + \log n)}{\log 4} = 24$$

$$(\log m + \log n) \left(\frac{1}{\log 8} + \frac{2}{\log 4} \right) = 24$$

$$\log mn = \frac{24}{\frac{1}{\log 8} + \frac{2}{\log 4}}$$

$$\log mn = \frac{24}{\frac{1}{3\log 2} + \frac{2}{2\log 2}} = \frac{12\log 2}{\frac{1}{3} + 1} = \frac{24\log 2}{\frac{4}{3}} = 18\log 2 = \log 2^{18}$$

$$mn = 2^{18}$$

Solusi 3:

$${}^8\log m + {}^4\log n^2 = 10$$

$${}^2\log \sqrt[3]{m} + {}^2\log n = 10$$

$${}^2\log n \sqrt[3]{m} = 10 \dots (1)$$

$${}^8\log n + {}^4\log m^2 = 14$$

$${}^2\log \sqrt[3]{n} + {}^2\log m = 15$$

$${}^2\log m \sqrt[3]{n} = 14 \dots (2)$$

Hasil penjumlahan persamaan (1) dan (2) adalah

$${}^2\log n \sqrt[3]{m} + {}^2\log m \sqrt[3]{n} = 10 + 14$$

$${}^2\log mn \sqrt[3]{mn} = 24$$

$$(mn)^{\frac{4}{3}} = 2^{24}$$

$$mn = 2^{24 \times \frac{3}{4}} = 2^{18}$$

56. Tentukan $({}^2\log x)^2$ jika ${}^2\log({}^8\log x) = {}^8\log({}^2\log x)$.

A. 243

B. 81

C. 27

D. 18

E. 9

Solusi: [C]

$${}^2\log({}^8\log x) = {}^8\log({}^2\log x)$$

$$8^{{}^2\log({}^8\log x)} = 8^{{}^8\log({}^2\log x)}$$

$$\left[2^{{}^2\log({}^8\log x)}\right]^3 = {}^2\log x$$

$$({}^8\log x)^3 = {}^2\log x$$

$$\left(\frac{{}^2\log x}{{}^2\log 8}\right)^3 = {}^2\log x$$

$$({}^2\log x)^2 = ({}^2\log 8)^3 = 3^3 = 27$$

57. Hitunglah $\sum_{n=1}^6 {}^2\log n$.

A. ${}^2\log 720$ B. ${}^2\log 640$ C. ${}^2\log 172$ D. ${}^2\log 520$ E. ${}^2\log 120$

Solusi: [A]

$$\sum_{n=1}^6 {}^2\log n = {}^2\log 1 + {}^2\log 2 + {}^2\log 3 + {}^2\log 4 + {}^2\log 5 + {}^2\log 6$$

$$= {}^2\log 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = {}^2\log 720$$

58. Selesaikanlah $10^{\frac{1}{8}\log 10} + (\sqrt{2})^{2\log 25} + 81^{3\log 2} = \dots$

A. 29 B. 21 C. 20 D. 19 E. 13

Solusi: [A]

$$10^{\frac{1}{8}\log 10} + (\sqrt{2})^{2\log 25} + 81^{3\log 2} = 10^{10\log 8} + (2^{2\log 25})^{\frac{1}{2}} + (3^{3\log 2})^4$$

$$= 8 + (25)^{\frac{1}{2}} + (2)^4 = 8 + 5 + 16 = 29$$

59. Sebuah lingkaran berjari-jari ${}^{12}\log m^6$ dan keliling ${}^{12}\log n^{12}$. Berapakah nilai ${}^m\log n$?

A. 2π B. π C. $\frac{1}{2\pi}$ D. $\frac{1}{\pi}$ E. $\frac{2}{\pi}$

Solusi: [D]

$$K = 2\pi r$$

$${}^{12}\log n^{12} = 2\pi {}^{12}\log m^6$$

$${}^2\log n^2 = 2\pi {}^2\log m$$

$$\frac{{}^2 \log n^2}{{}^2 \log m} = 2\pi$$

$$\frac{2 {}^2 \log n}{{}^2 \log m} = 2\pi$$

$${}^m \log n = \pi$$

60. Diberikan $a = {}^2 \log(\sqrt{2} + \sqrt{3} + 2) + {}^2 \log(\sqrt{2} - \sqrt{3} + 2)$ dan

$$b = {}^2 \log(-\sqrt{2} + \sqrt{3} + 2) + {}^2 \log(\sqrt{2} + \sqrt{3} - 2)$$

Nilai dari $(a + b)^{23} \log 1024 = \dots$

A. 7

B. 8

C. 9

D. 10

E. 12

Solusi: [D]

$$a = {}^2 \log(\sqrt{2} + \sqrt{3} + 2) + {}^2 \log(\sqrt{2} - \sqrt{3} + 2)$$

$$= {}^2 \log(\sqrt{2} + \sqrt{3} + 2)(\sqrt{2} - \sqrt{3} + 2) = {}^2 \log\left[(\sqrt{2} + 2)^2 - (\sqrt{3})^2\right]$$

$$= {}^2 \log(6 + 4\sqrt{2} - 3) = {}^2 \log(3 + 4\sqrt{2})$$

$$b = {}^2 \log(-\sqrt{2} + \sqrt{3} + 2) + {}^2 \log(\sqrt{2} + \sqrt{3} - 2)$$

$$= {}^2 \log\left[(\sqrt{3})^2 - (\sqrt{2} - 2)^2\right]$$

$$= {}^2 \log(3 - 6 + 4\sqrt{2}) = {}^2 \log(4\sqrt{2} - 3)$$

$$a + b = {}^2 \log(3 + 4\sqrt{2}) + {}^2 \log(4\sqrt{2} - 3) = {}^2 \log(4\sqrt{2} + 3)(4\sqrt{2} - 3)$$

$$= {}^2 \log(32 - 9) = {}^2 \log 23$$

$$\therefore (a + b)^{23} \log 1024 = {}^2 \log 23 \cdot {}^{23} \log 1024 = {}^2 \log 1024 = 10$$