

# PENGAYAAN MATEMATIKA

## SOLUSI GEOMETRI 5

1. COMC, 2000

Solve the equation  $4(16^{\sin^2 x}) = 2^{6 \sin x}$ , for  $0 \leq x \leq 2\pi$ .

**Solusi:**

$$4(16^{\sin^2 x}) = 2^{6 \sin x}$$

$$2^2(2^{4 \sin^2 x}) = 2^{6 \sin x}$$

$$2^{2+4 \sin^2 x} = 2^{6 \sin x}$$

$$2 + 4 \sin^2 x = 6 \sin x$$

$$2 \sin^2 x - 3 \sin x + 1 = 0$$

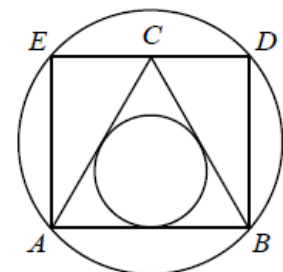
$$(2 \sin x - 1)(\sin x - 1) = 0$$

$$\sin x = \frac{1}{2} \text{ atau } \sin x = 1$$

$$\text{Jadi, } x = \frac{\pi}{6}, x = \frac{5\pi}{6}, \text{ atau } x = \frac{\pi}{2}$$

2. COMC, 2000

In the diagram,  $\triangle ABC$  is equilateral and the radius of its inscribed circle is 1. A larger circle is drawn through the vertices of the rectangle  $ABDE$ . What is the diameter of the larger circle?



**Solusi:**

Karena  $\triangle ABC$  sama sisi, maka sudutnya adalah  $60^\circ$ .

$O$  adalah titik pusat lingkaran dalam dengan jari-jari  $OP = 1$ , sehingga  $OB$  bertindak sebagai garis bagi sudut  $B$ .

$$BP = \frac{OP}{\tan 30^\circ} = \frac{1}{\frac{1}{3}\sqrt{3}} = \sqrt{3}$$

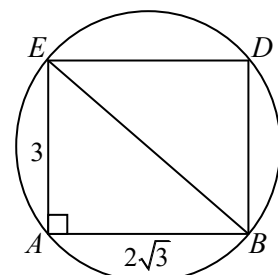
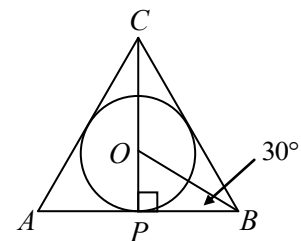
$$AB = 2BP = 2\sqrt{3}$$

$$CP = BP \tan 60^\circ = \sqrt{3} \times \sqrt{3} = 3$$

$$AE = BD = CP = 3$$

$$BE = \sqrt{(2\sqrt{3})^2 + 3^2} = \sqrt{21}$$

Jadi, diameter lingkaran terbesar adalah  $\sqrt{21}$ .



3. COMC, 2000

Triangle  $ABC$  has vertices  $A(0, 0)$ ,  $B(9, 0)$  and  $C(0, 6)$ . The points  $P$  and  $Q$  lie on side  $AB$  such that  $AP = PQ = QB$ . Similarly, the points  $R$  and  $S$  lie on side  $AC$  so that  $AR = RS = SC$ .

The vertex  $C$  is joined to each of the points  $P$  and  $Q$ . In the same way,  $B$  is joined to  $R$  and  $S$ .

- Determine the equation of the line through the points  $R$  and  $B$ .
- Determine the equation of the line through the points  $P$  and  $C$ .
- The line segments  $PC$  and  $RB$  intersect at  $X$ , and the line segments  $QC$  and  $SB$  intersect at  $Y$ . Prove that the points  $A$ ,  $X$  and  $Y$  lie on the same straight line.

**Solusi:**

- (a) Koordinat titik  $B$  adalah  $(9, 0)$ .

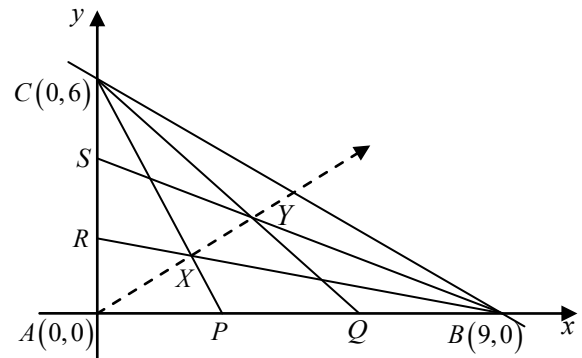
$$\text{Gradien garis } RB : m_{RB} = \frac{2-0}{0-9} = -\frac{2}{9}$$

$$\text{Persamaan garis } RB: y = -\frac{2}{9}x + 2$$

- (b) Koordinat titik  $P$  adalah  $(3, 0)$ .

$$\text{Gradien garis } PC : m_{PC} = \frac{6-0}{0-3} = -2$$

$$\text{Persamaan garis } PC: y = -2x + 6$$



- (c) Koordinat titik  $X$  adalah perpotongan garis  $RB \equiv y = -\frac{2}{9}x + 2$  dan  $PC \equiv y = -2x + 6$ .

$$-\frac{2}{9}x + 2 = -2x + 6$$

$$\frac{16}{9}x = 4$$

$$x = \frac{9}{4}$$

$$y = -2 \cdot \frac{9}{4} + 6 = \frac{3}{2}$$

Koordinat titik  $X$  adalah  $\left(\frac{9}{4}, \frac{3}{2}\right)$

$$\text{Gradien garis } QC : m_{QC} = \frac{6-0}{0-6} = -1$$

$$\text{Persamaan garis } QC: y = -x + 6$$

$$\text{Gradien garis } SB : m_{SB} = \frac{4-0}{0-9} = -\frac{4}{9}$$

$$\text{Persamaan garis } SB: y = -\frac{4}{9}x + 4$$

Koordinat titik  $Y$  adalah perpotongan garis  $QC \equiv y = -x + 6$  dan  $SB \equiv y = -\frac{4}{9}x + 4$ .

$$-\frac{4}{9}x + 4 = -x + 6$$

$$\frac{5}{9}x = 2$$

$$x = \frac{18}{5}$$

$$y = -\frac{18}{5} + 6 = \frac{12}{5}$$

Koordinat titik  $Y$  adalah  $\left(\frac{18}{5}, \frac{12}{5}\right)$ .

Persamaan garis yang melalui titik-titik  $(0,0)$  dan  $\left(\frac{9}{4}, \frac{3}{2}\right)$  adalah  $y = \frac{2}{3}x$ .

Titik  $Y\left(\frac{18}{5}, \frac{12}{5}\right)$  harus dilalui garis  $y = \frac{2}{3}x$ , sehingga

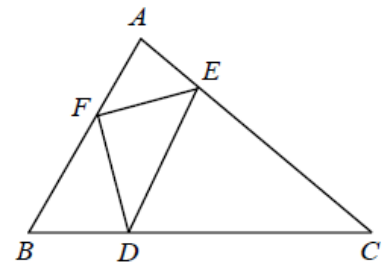
$$\frac{12}{5} = \frac{2}{3} \times \frac{18}{5} \text{ (pernyataan yang bernilai benar)}$$

Jadi, terbukti bahwa titik-titik  $A, X,$  dan  $Y$  terletak pada sebuah garis.

#### 4. COMC, 2000

In  $\triangle ABC$ , the points  $D, E$  and  $F$  are on sides  $BC, CA$  and  $AB$ , respectively, such that  $\angle AFE = \angle BFD$ ,  $\angle BDF = \angle CDE$ , and  $\angle CED = \angle AEF$ .

- (a) Prove that  $\angle BDF = \angle BAC$ .  
 (b) If  $AB = 5, BC = 8$  and  $CA = 7$ , determine the length of  $BD$ .



#### Solusi:

- (a) Misalnya  $\angle AFE = \angle BFD = x$ ,  $\angle BDF = \angle CDE = y$ , dan

$$\angle CED = \angle AEF = z.$$

$$\angle ACB = 180^\circ - (y + z)$$

$$\angle ABC = 180^\circ - (x + y)$$

$$\angle BAC = 180^\circ - 180^\circ + (y + z) - 180^\circ + (x + y) = 180^\circ - (x + z)$$

$$\angle BAC = x + 2y + z - 180^\circ = 180^\circ - (x + z)$$

$$2x + 2y + 2z = 360^\circ$$

$$x + y + z = 180^\circ$$

$$y = 180^\circ - (x + z) = \angle BAC$$

Jadi,  $\angle BDF = \angle BAC$  (QED)

- (b) Dari butir (a) diperoleh  $\angle A = y, \angle B = z,$  dan  $\angle C = x$

Dengan sudut-sudut yang sama diperoleh bahwa

$\triangle ABC \sim \triangle DBF \sim \triangle DEC \sim \triangle AEF$ , sehingga

$$\frac{BD}{BF} = \frac{BA}{BC} = \frac{5}{8}, \frac{CD}{CE} = \frac{CA}{CB} = \frac{7}{8}, \frac{AE}{AF} = \frac{AB}{AC} = \frac{5}{7}$$

Misalnya  $BD = 5k$  dan  $BF = 8k$ ,  $CD = 7l$  dan  $CE = 8l$ ,  $AE = 5m$  dan  $AF = 7m$ .

$$7m + 8k = 5 \dots (1)$$

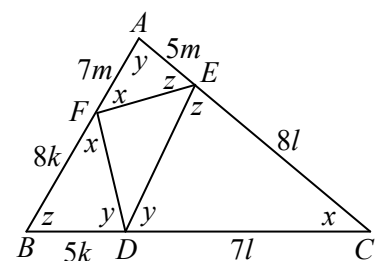
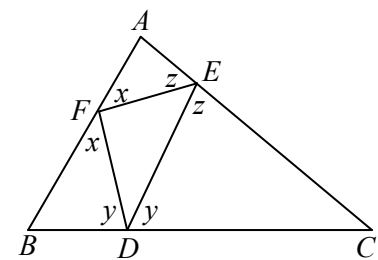
$$5k + 7l = 8 \dots (2)$$

$$5m + 8l = 7 \dots (3)$$

$8 \times$  Persamaan (2)  $- 7 \times$  Persamaan (3) menghasilkan

$$40k - 35m = 15 \dots (4)$$

$5 \times$  Persamaan (1)  $+ 7 \times$  Persamaan (4) menghasilkan



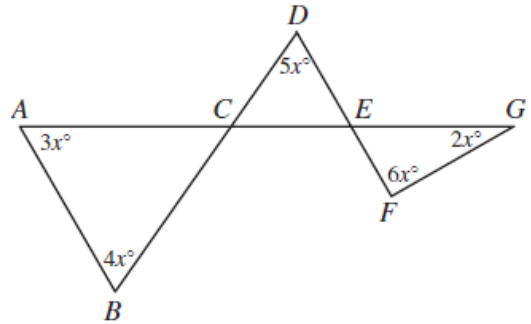
$$80k = 40$$

$$k = \frac{1}{2}$$

Jadi, panjang  $BD = 5k = \frac{5}{2}$ .

5. COMC, 2001

In the given diagram, what is the value of  $x$ ?



**Solusi:**

$$\angle DCE = 180^\circ - (3x + 4x)^\circ = 180^\circ - 7x^\circ$$

$$\angle DEC = 180^\circ - (2x + 6x)^\circ = 180^\circ - 8x^\circ$$

$$\angle DCE + \angle DEC + \angle CDE = 180^\circ - 7x^\circ + 180^\circ - 8x^\circ + 5x^\circ = 180^\circ$$

$$10x^\circ = 180^\circ$$

$$x = 18$$