PENGAYAAN MATEMATIKA
SOLUSI GEOMETRI 4

1. COMC, 1999

Determine all x which satisfy:

$$2\sin^3 x + 6\sin^2 x - \sin x - 3 = 0, 0 < x < 2\pi$$

Solusi:

$2\sin^3 x + 6\sin^2 x - \sin x - 3 = 0$

$2\sin^2 x(\sin x + 3) - (\sin x + 3) = 0$

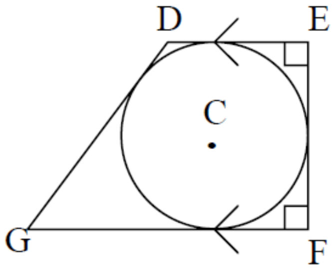
$(2\sin^2 x - 1)(\sin x + 3) = 0$

$\sin x = \pm\frac{1}{2}\sqrt{2}$ (diterima) atau $\sin x = -3$ (ditolak)

Jadi, $x = \frac{\pi}{4}, x = \frac{3\pi}{4}, x = \frac{5\pi}{4}$, atau $x = \frac{7\pi}{4}$

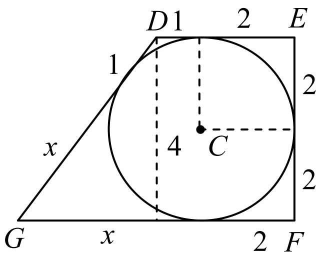
2. COMC, 1999

A trapezoid, $DEFG$, is circumscribed about a circle that has centre C and radius 2, as is shown. The shorter of the two parallel sides, DE, has length 3 and angles DEF and EFG are right angles. Determine the area of the trapezoid.



Solusi:

Menurut Pythagoras:



$(x+1)^2 = 4^2 + (x-1)^2$

$x^2 + 2x + 1 = 16 + x^2 - 2x + 1$

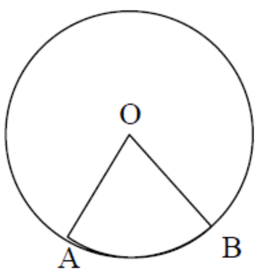
$4x = 16$

$x = 4$

$[DEFG] = \frac{1}{2}(6+3)4 = 18$

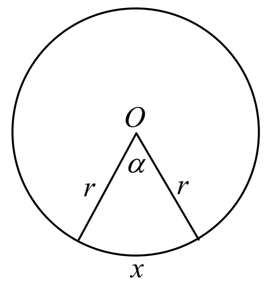
3. COMC, 1999

The sector OAB of a circle, with centre O, has a perimeter of 12. Determine the radius of the circle which maximizes the area of the sector.



Solusi:

Keliling sector OAB $=2r+x=12$



$x=12-2r$

Panjang busur $x=\frac{\alpha}{2\pi}\times 2\pi r=\alpha r$

$12-2r=\alpha r$

$\frac{12-2r}{r}=\alpha$

Luas sector $L=\frac{\alpha}{2\pi}\times\pi r^2=\frac{1}{2}\alpha r^2=\frac{1}{2}\alpha r^2=\frac{1}{2}\times\frac{12-2r}{r}\times r^2=6r-r^2$

$L'=6-2r=0$

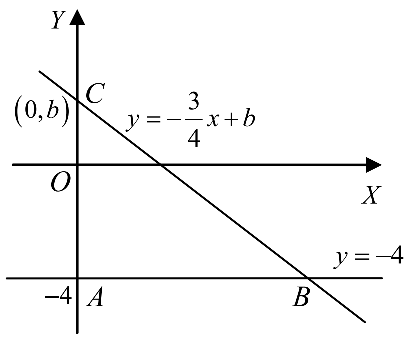
$r=3$

4. COMC, 1999

Two identical triangles each have an area of 24. Their vertices are determined by the intersection of the lines with equations $y=-4, x=0$ and $y=\frac{-3}{4}x+b$. Determine the two possible values for b.

Solusi:

Gradien garis $y=-\frac{3}{4}x+b$ adalah $m=-\frac{3}{4}$.



Misalnya $AB=4a$ dan $AC=3a$.

$[ABC]=\frac{1}{2}\times 3a\times 4a=24$

$a^2=4$

$a=\pm 2$

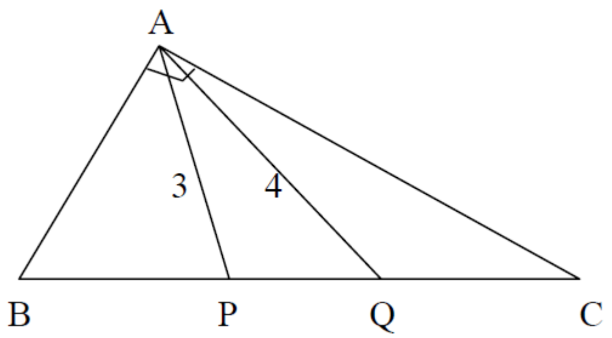
$3a=\pm 6$

$b+4=\pm 6$

$b=2$ atau $b=-10$

5. COMC, 1999

Triangle ABC is right angled with its right angle at A. The points P and Q are on the hypotenuse BC such that $BP=PQ=QC$, $AP=3$ and $AQ=4$. Determine the length of each side of $\triangle ABC$.



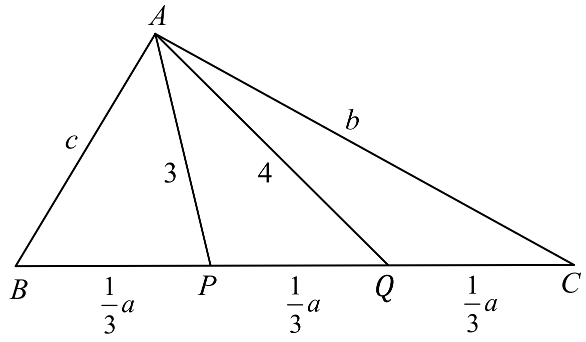
Solusi:

Karena ΔABC siku-siku di A, maka $\cos B=\frac{c}{a}$ dan $\cos C=\frac{b}{a}$.

Menurut aturan Kosinus dalam ΔBPA dan ΔCQA,

$3^2=c^2+\left(\frac{1}{3}a\right)^2-2c\cdot\frac{1}{3}a\cos B$

$9 = c^2 + \frac{1}{9}a^2 - 2c \cdot \frac{1}{3}a \cdot \frac{c}{a}$



$9 = c^2 + \frac{1}{9}a^2 - \frac{2}{3}c^2$

$9 = \frac{1}{3}c^2 + \frac{1}{9}a^2$

$81 = 3c^2 + a^2$ .... (1)

$4^2 = b^2 + \left(\frac{1}{3}a\right)^2 - 2b \cdot \frac{1}{3}a \cos C$

$16 = b^2 + \frac{1}{9}a^2 - 2b \cdot \frac{1}{3}a \cdot \frac{b}{a}$

$16 = b^2 + \frac{1}{9}a^2 - \frac{2}{3}b^2$

$16 = \frac{1}{3}b^2 + \frac{1}{9}a^2$

$144 = 3b^2 + a^2$ .... (2)

Penjumlahan persamaan (1) dan (2) menghasilkan

$225 = 3\left(b^2 + c^2\right) + 2a^2$

$225 = 3a^2 + 2a^2$

$5a^2 = 225$

$a^2 = 45$

$a = \pm\sqrt{45}$

Karena $a > 0$, maka $a = \sqrt{45} = 3\sqrt{5}$

Substitusikan $a = \sqrt{45}$ ke persamaan (1) dan (2) sehingga diperoleh

$81 = 3c^2 + \left(\sqrt{45}\right)^2$

$3c^2 = 36$

$c^2 = 12$

$c = \sqrt{12} = 2\sqrt{3}$

$144 = 3b^2 + \left(\sqrt{45}\right)^2$

$3b^2 = 99$

$b^2 = 33$

$b = \sqrt{33}$