Solusi Pengayaan Matematika
Edisi 4

Nomor Soal: 31-40

31. Bentuk sederhana dari

$$\frac{1}{{}^{2}\log 2000!}+\frac{1}{{}^{3}\log 2000!}+\frac{1}{{}^{4}\log 2000!}+...+\frac{1}{{}^{2000}\log 20!}$$ adalah ….

A. 0 B. 1 C. 10 D. 2000 E. 2000!

Solusi: [B]

$$\frac{1}{{}^{2}\log 2000!}+\frac{1}{{}^{3}\log 2000!}+\frac{1}{{}^{4}\log 2000!}+...+\frac{1}{{}^{2000}\log 20!}$$

$$={}^{2000!}\log 2+{}^{2000!}\log 3+{}^{2000!}\log 4+{}^{2000!}\log 5+...+{}^{20!}\log 2000$$

$$={}^{2000!}\log 2\times 3\times 4\times 5...\times 2000$$

$$={}^{2000!}\log 1\times 2\times 3\times 4\times 5...\times 2000={}^{2000!}\log 2000!=1$$

32. Tentukan nilai dari ab jika ${}^{8}\log a^{3}+{}^{4}\log b^{6}=7$ dan ${}^{2}\log b^{5}-{}^{\frac{1}{2}}\log a^{3}=13$.

A. 3 B. 8 C. 16 D. 32 E. 64

Solusi: [B]

$${}^{8}\log a^{3}+{}^{4}\log b^{6}=7$$

$${}^{2}\log a+{}^{2}\log b^{3}=7$$

$${}^{2}\log ab^{3}=7$$

$ab^{3}=2^{7}$ …. (1)

$${}^{2}\log b^{5}-{}^{\frac{1}{2}}\log a^{3}=13$$

$${}^{2}\log b^{5}+{}^{2}\log a^{3}=13$$

$${}^{2}\log a^{3}b^{5}=13$$

$a^{3}b^{5}=2^{13}$ …. (2)

Dari persamaan (1) dan (2) didapat

$$\left(ab^{3}\right)a^{2}b^{2}=2^{13}$$

$$2^{7}a^{2}b^{2}=2^{13}$$

$$a^{2}b^{2}=2^{6}$$

$$ab=2^{3}=8$$

33. Jika $2010^{\log 2009} = x^{\log 2010}$, maka $x = ...$

A. $\frac{2010}{2009}$ B. ${}^{2009}\log 2010$ C. 2008 D. 2009 E. 2010

Solusi: [D]

$$2010^{\log 2009} = x^{\log 2010}$$

$$\log 2010^{\log 2009} = \log x^{\log 2010}$$

$$\log 2009 \log 2010 = \log 2010 \log x$$

$$\log 2009 = \log x$$

$$x = 2009$$

34. Berapakah hasil kali dari solusi: ${}^{2187}\log x - {}^{x}\log 9 = \frac{5}{7}$.

A. $\frac{1}{9}$ B. 3 C. 27 D. 243 E. 729

Solusi: [D]

$${}^{2187}\log x - {}^{x}\log 9 = \frac{5}{7}$$

$$\frac{{}^{3}\log x}{{}^{3}\log 2187} - \frac{{}^{3}\log 9}{{}^{3}\log x} = \frac{5}{7}$$

$$\frac{{}^{3}\log x}{7} - \frac{2}{{}^{3}\log x} = \frac{5}{7}$$

$$\left({}^{3}\log x\right)^2 - 14 = 5\,{}^{3}\log x$$

$$\left({}^{3}\log x\right)^2 - 5\,{}^{3}\log x - 14 = 0$$

$$\left({}^{3}\log x - 7\right)\left({}^{3}\log x + 2\right) = 0$$

$${}^{3}\log x = 7 \vee {}^{3}\log x = -2$$

$$x_1 = 3^7 \vee x_2 = 3^{-2}$$

$$x_1 x_2 = 3^7 \cdot 3^{-2} = 3^5 = 243$$

35. Tentukan ${}^{16}\log \sum_{r=0}^{64} \frac{64!}{(64-r)!r!}$.

A. 4 B. 8 C. 16 D. 32 E. 64

Solusi: [C]

Pikirkan Segitiga Pascal:

$$\sum_{r=0}^{64} \frac{64!}{(64-r)!r!} = \sum_{r=0}^{64} {}_{64}C_r = (1+1)^{64} = 2^{64} = \left(2^4\right)^{16} = 16^{16}$$

$$^{16}\log\sum_{r=0}^{64}\frac{64!}{(64-r)!r!}={}^{16}\log 16^{16}=16$$

36. Berapakah nilai dari $4^{\left(\log 10^{10^6}-1002\times 998\right)}$?

A. 256 B. 128 C. 64 D. 32 E. 16

Solusi: [A]

$$\log 10^{10^6}-1002\times 998=10^6-\left(10^3+2\right)\left(10^3-2\right)=10^6-10^6+4=4$$

Jadi, $4^{\left(\log 10^{10^6}-1002\times 998\right)}=4^4=256$

37. Diberikan $\log(\log(\log(x-1)+8))=0$. Tentukan banyak faktor dari 2009^{108-x}.

A. 1024 B. 512 C. 120 D. 80 E. 64

Solusi: [C]

$$\log(\log(\log(x-1)+8))=0$$
$$\log(\log(x-1)+8)=1$$
$$\log(x-1)+8=10$$
$$\log(x-1)=2$$
$$x-1=100$$
$$x=101$$
$$2009^{108-x}=\left(7^2\cdot 41\right)^{108-101}=7^{14}\cdot 41^7$$

Jadi, banhyak faktornya adalah $\left(14+1\right)\left(7+1\right)=120$

38. Berapakah nilai dari $2009\left\lfloor\frac{\left(\ln\pi\right)\left({}^2\log e\right)}{{}^2\log 10}\right\rfloor$, di mana $\lfloor x\rfloor$ mewakili bilangan bulat terbesar kurang dari atau sama dengan *x*?

A. −2 B. −1 C. 0 D. 1 E. 2

Solusi: [C]

$$\frac{\left(\ln\pi\right)\left({}^2\log e\right)}{{}^2\log 10}=\frac{\frac{\log\pi}{\log e}\times\frac{\log e}{\log 2}}{\frac{\log 10}{\log 2}}=\frac{\frac{\log\pi}{\log 2}}{\frac{1}{\log 2}}=\log\pi$$

$$2009\left\lfloor\frac{\left(\ln\pi\right)\left({}^2\log e\right)}{{}^2\log 10}\right\rfloor=2009\log\lfloor\pi\rfloor$$

Karena $10^0<\pi<10^1$, $0<\log\pi<1$, maka $\lfloor\log\pi\rfloor=0$.

Jadi, $2009\left\lfloor \dfrac{(\ln\pi)\left({}^{2}\log e\right)}{{}^{2}\log 10} \right\rfloor = 2009\log\lfloor \pi \rfloor = 2009\cdot 0 = 0$

39. Sederhanakan $\dfrac{1}{\dfrac{1}{{}^{7}\log 2009}+\dfrac{1}{{}^{41}\log 2009}+\dfrac{1}{{}^{14}\log 2009}-\dfrac{1}{{}^{2}\log 2009}}$.

A. 1 B. 2 C. 3 D. 4 E. 5

Solusi: [A]

$$\frac{1}{\dfrac{1}{{}^{7}\log 2009}+\dfrac{1}{{}^{41}\log 2009}+\dfrac{1}{{}^{14}\log 2009}-\dfrac{1}{{}^{2}\log 2009}}$$

$$= \frac{1}{{}^{2009}\log 7 + {}^{2009}\log 41 + {}^{2009}\log 14 - {}^{2009}\log 2} = \frac{1}{{}^{2009}\log \dfrac{7\cdot 41\cdot 14}{2}}$$

$$= \frac{1}{{}^{2009}\log 2009} = \frac{1}{1} = 1$$

40. Jika $e^{4x}+e^{-4x}=47$, berapakah nilai dari $\dfrac{2009}{{}^{324}\log\left(e^{3x}+e^{-3x}\right)}$, dengan x adalah bilangan real?

A. 252 B. 504 C. 1004,5 D. 2009 E. 4018

Solusi: [E]

$$e^{4x}+e^{-4x}=47$$

$$\left(e^{2x}+e^{-2x}\right)^2-2=47$$

$$\left(e^{2x}+e^{-2x}\right)^2=49$$

$$e^{2x}+e^{-2x}=7$$

$$\left(e^{x}+e^{-x}\right)^2-2=7$$

$$\left(e^{x}+e^{-x}\right)^2=9$$

$$e^{x}+e^{-x}=3$$

$$\therefore e^{3x}+e^{x}+e^{-x}+e^{-3x}=\left(e^{x}+e^{-x}\right)\left(e^{2x}+e^{-2x}\right)=3\cdot 7=21$$

$$\therefore e^{3x}+e^{-3x}=21-\left(e^{x}+e^{-x}\right)=21-3=18$$

$$\therefore \frac{2009}{{}^{324}\log\left(e^{3x}+e^{-3x}\right)}=\frac{2009}{{}^{324}\log 18}=\frac{2009}{\dfrac{1}{2}}=4018$$