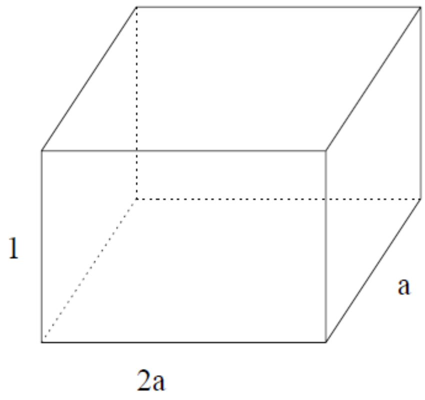
PENGAYAAN MATEMATIKA

SOLUSI GEOMETRI 3

1. COMC,1998

A rectangular closed box (shown) with dimensions a, $2a$ and 1 has a surface area of 54, where a is an integer. Determine the volume of the box.



Solusi:

Luas permukaan $= 2(2a \cdot a + 2a \cdot 1 + a \cdot 1) = 54$

$2a^2 + 3a - 27 = 0$

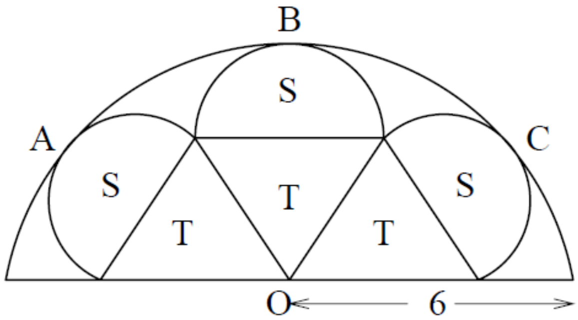
$(2a+9)(a-3) = 0$

$a = -\frac{9}{2}$ (ditolak) ataua $= 3$ (diterima)

Volume kotak $= 2a \cdot a \cdot 1 = 2a^2 = 2 \cdot 3^2 = 18$

2. COMC, 1998

In the figure, each region T represents an equilateral triangle and each region S a semicircle. The complete figure is a semicircle of radius 6 with its centre O. The three smaller semicircles touch the large semicircle at points A, B and C. What is the radius of a semicircle S?



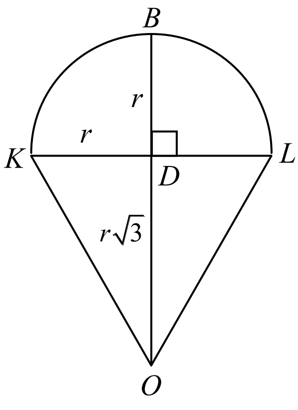
Solusi:

Karena KL adalah diameter, maka jari-jarinya $DK = DL = r$.

Karena ΔOKL sama sisi, maka garis tinggi $OD = r\sqrt{3}$.

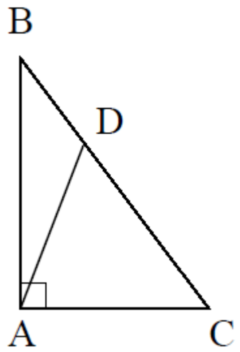
$OB = 6 = r + r\sqrt{3}$

$r = \frac{6}{1+\sqrt{3}} = 3\left(\sqrt{3} - 1\right)$



3. COMC. 1998

The lengths of the sides of triangle ABC are 60, 80 and 100 with $\angle A = 90°$. The line AD divides triangle ABC into two triangles of equal perimeter. Calculate the length of AD.



Solusi:

Keliling ΔACD = keliling ΔADB

$AC + CD + AD = AB + BD + AD$

$60 + CD + AD = 80 + BD + AD$

$CD - BD = 20$ .... (1)

$CD + BD = 100$ .... (2)

Dari persamaan (1) dan (2) diperoleh $CD = 60$ dan $BD = 40$.

$\cos C = \frac{60}{100} = \frac{3}{5}$

Menurut aturan Kosinus dalam ΔACD

$AD^2 = AC^2 + CD^2 - 2AC \cdot CD \cos C$

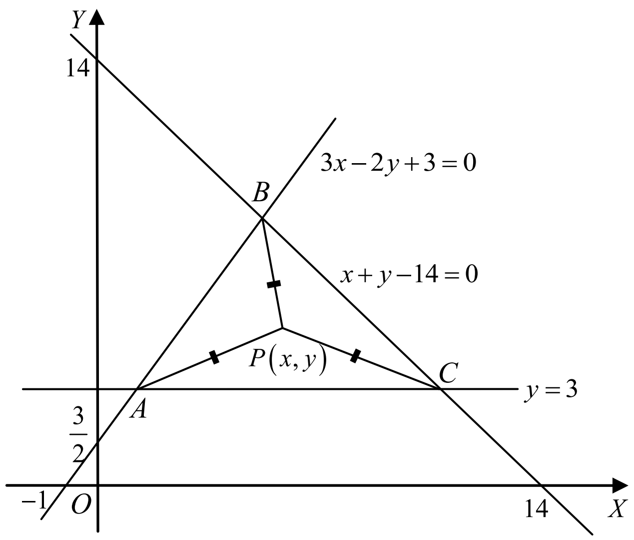
$AD^2 = 60^2 + 60^2 - 2 \cdot 60 \cdot 60 \cdot \frac{3}{5} = 2880$

$AD = \sqrt{2880} = 24\sqrt{5}$

4. COMC, 1998

Triangle ABC has its sides determinded in the following way: side AB by line $3x - 2y + 3 = 0$; side BC by line $x + y - 14 = 0$; and side AC by line $y = 3$. If the point P is chosen so that $PA = PB = PC$, determine the equation of the line containing A and P.

Solusi:



$y = 3 \rightarrow 3x - 2y + 3 = 0$

$3x - 2 \cdot 3 + 3 = 0$

$x = 1$

Koordintat titik A adalah $(1,3)$.

$y = 3 \rightarrow x + y - 14 = 0$

$x + 3 - 14 = 0$

$x = 11$

Koordintat titik B adalah $(11,3)$.

$y = 14 - x \rightarrow 3x - 2y + 3 = 0$

$3x - 2(14 - x) + 3 = 0$

$x = 5$

$y = 14 - 5 = 9$

Koordintat titik C adalah $(5,9)$.

Koordinat titik tengah AC adalah $\left(\frac{1}{2}(1+11),\frac{1}{2}(3+3)\right)=(6,3)$.

Persamaan garis sumbu AC adalah $x=6$.

Koordinat titik tengah AB adalah $\left(\frac{1}{2}(1+5),\frac{1}{2}(3+9)\right)=(3,6)$.

Gradien garis AB adalah $m_1=\frac{9-3}{5-1}=\frac{6}{4}=\frac{3}{2}$

Gradien garis sumbu AB adalah $m_2=-\frac{1}{m_1}=-\frac{2}{3}$

Persamaan garis sumbu AB adalah $y-6=-\frac{2}{3}(x-3)$

Perpotongan garis sumbu AC dan AB adalah koordinat titik P, sehingga

$$x=6\rightarrow y-6=-\frac{2}{3}(x-3)$$

$$y-6=-\frac{2}{3}(6-3)$$

$$y=4$$

Koordinat titik P adalah $(6,4)$.

Gradien AP adalah $m_{AP}=\frac{4-3}{6-1}=\frac{1}{5}$

Persamaan garis yang melalui A dan P adalah

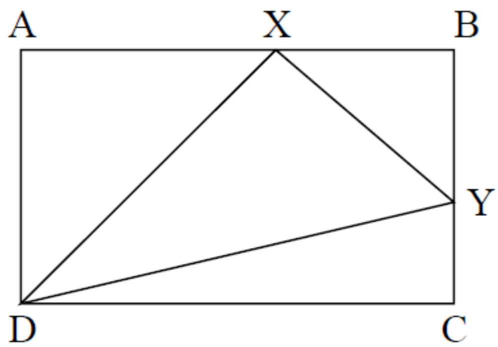
$$y-3=\frac{1}{5}(x-1)$$

$$5y-15=x-1$$

$$x-5y+14=0$$

5. COMC, 1998

$ABCD$ is a rectangle and lines DX, DY and XY are drawn where X is on AB and Y is on BC. The area of triangle AXD is 5, the area of triangle BXY is 4 and the area of triangle CYD is 3. Determine the area of triangle DXY.



Solusi:

Misalnya ukuran persegi panjang $ABCD$ adalah $AD=x$ dan $CD=y$.

$$[CYD]=\frac{1}{2}y\cdot CY=3$$

$$CY=\frac{6}{y}$$

$$BY=x-CY=x-\frac{6}{y}=\frac{xy-6}{y}$$

$$[AXD] = \frac{1}{2} x \cdot AX = 5$$

$$AX = \frac{10}{x}$$

$$BX = y - AX = y - \frac{10}{x} = \frac{xy - 10}{x}$$

$$[BXY] = \frac{1}{2} BY \cdot BX = \frac{1}{2} \cdot \frac{xy - 6}{y} \cdot \frac{xy - 10}{x} = 4$$

$$(xy)^2 - 16xy + 60 = 8xy$$

$$(xy)^2 - 24xy + 60 = 0$$

$$xy = \frac{24 \pm \sqrt{576 - 240}}{2} = \frac{24 \pm 4\sqrt{21}}{2} = 12 \pm 2\sqrt{21}$$

Luas persegi panjang adalah $xy = 12 + 2\sqrt{21}$

$$[DXY] = [ABCD] - [AXD] - [BXY] - [CYD] = 12 + 2\sqrt{21} - 5 - 4 - 3 = 2\sqrt{21}$$