Solusi Pengayaan Matematika
Edisi 3

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21. Carilah himpunan penyelesaian dari sistem persamaan $\begin{cases} x = 16y \\ {}^{y}\log x - {}^{x}\log y = \frac{8}{3} \end{cases}$

Solusi:

$x = 16y$

${}^{y}\log x - {}^{x}\log y = \frac{8}{3}$

$\frac{\log x}{\log y} + \frac{\log y}{\log x} = \frac{8}{3}$ …. (1)

Misalnya $\frac{\log x}{\log y} = a$, maka persamaan (1) menjadi:

$a + \frac{1}{a} = \frac{8}{3}$

$3a^2 - 8a - 3 = 0$

$(3a+1)(a-3) = 0$

$a = -\frac{1}{3}$ atau $a = 3$

$\left.\begin{matrix} a = -\frac{1}{3} \\ x = 16y \end{matrix}\right\} \to \frac{\log x}{\log y} = a$ atau $\left.\begin{matrix} a = 3 \\ x = 16y \end{matrix}\right\} \to \frac{\log x}{\log y} = a$

$\frac{\log 16y}{\log y} = -\frac{1}{3}$ atau $\frac{\log 16y}{\log y} = 3$

$3\log 16y = -\log y$ atau $\log 16y = 3\log y$

$\log(16y)^3 = \log y^{-1}$ atau $\log 16y = \log y^3$

$(16y)^3 = y^{-1}$ atau $16y = y^3$

$4^6 y^3 = \frac{1}{y}$ atau $y^3 - 16y = 0$

$4^6 y^4 = 1$ atau $y(y^2 - 16) = 0$

$(4^3 y^2 + 1)(4^3 y^2 - 1) = 0$ atau $y(y+4)(y-4) = 0$

$(64y^2 + 1)(8y+1)(8y-1) = 0$ atau $y = 0$, ± 4

$y = \pm\frac{1}{8}$ atau $y = 0$, ± 4

$y = \frac{1}{8}$ atau $y = 4$ (karena $x > 0$ dan $y > 0$)

$y = \frac{1}{8} \rightarrow x = 16y = 16\left(\frac{1}{8}\right) = 2$ atau $y = 4 \rightarrow x = 16y = 16(4) = 64$

Jadi, himpunan penyelesaiannya adalah $\left\{\left(2, \frac{1}{8}\right), (64, 4)\right\}$

22. Selesaikan sistem persamaan $\begin{cases} \log(2xy) = \log x \log y \\ \log(yz) = \log y \log z \\ \log(2zx) = \log z \log x \end{cases}$.

Solusi:

Persamaan (1) dijabarkan sebagai berikut.

$\log(2xy) = \log x \log y$

$\log x \log y + 1 = \log 2 + \log x + \log y + 1$ (kedua ruas ditambah 1)

$\log x \log y - \log x - \log y + 1 = \log 2 + 1$

$\log x(\log y - 1) - (\log y - 1) = \log 20$

$(\log x - 1)(\log y - 1) = \log 20$

Persamaan (2) dijabarkan sebagai berikut.

$\log(yz) = \log y \log z$

$\log y \log z + 1 = \log y + \log z + 1$ (kedua ruas ditambah 1)

$\log y \log z - \log y - \log z + 1 = 1$

$\log z(\log y - 1) - (\log y - 1) = 1$

$(\log y - 1)(\log z - 1) = 1$

Persamaan (3) analog dengan persamaan (2), sehingga penjabarannya adalah

$(\log x - 1)(\log z - 1) = \log 20$

Sehingga sistem persamaan semula identik dengan sistem persamaan berikut ini.

$$\begin{cases} (\log x - 1)(\log y - 1) = \log 20 \\ (\log y - 1)(\log z - 1) = 1 \\ (\log x - 1)(\log z - 1) = \log 20 \end{cases}$$

Dengan mengalikan ketiga persamaan itu diperoleh

$\{(\log x - 1)(\log y - 1)(\log z - 1)\}^2 = \log^2 20$

$(\log x - 1)(\log y - 1)(\log z - 1) = \pm \log 20$

$(\log x - 1)(\log y - 1) = \log 20 \rightarrow (\log x - 1)(\log y - 1)(\log z - 1) = \pm \log 20$

$\log 20(\log z - 1) = \pm \log 20$

$$\log z = 1 \pm 1$$

$$z = 100 \text{ atau } z = 1$$

$$(\log y - 1)(\log z - 1) = 1 \rightarrow (\log x - 1)(\log y - 1)(\log z - 1) = \pm \log 20$$

$$(\log x - 1)1 = \pm \log 20$$

$$\log x = 1 \pm \log 20$$

$$x = 200 \text{ atau } x = \frac{1}{2}$$

$$(\log x - 1)(\log z - 1) = \log 20 \rightarrow (\log x - 1)(\log y - 1)(\log z - 1) = \pm \log 20$$

$$(\log y - 1)\log 20 = \pm \log 20$$

$$\log y = 1 \pm 1$$

$$y = 100 \text{ atau } y = 1$$

Jadi, penyelesaian sistem persamaan itu adalah $\{(200,100,100)\}$ atau $\left\{\left(\frac{1}{2},1,1\right)\right\}$.

23. Carilah himpunan solusi dari sistem persamaan $\begin{cases} {}^2\log x + {}^4\log y + {}^4\log z = 2 \\ {}^3\log y + {}^9\log z + {}^9\log x = 2 \\ {}^4\log z + {}^{16}\log x + {}^{16}\log y = 2 \end{cases}$.

Solusi:

$${}^2\log x + {}^4\log y + {}^4\log z = 2$$

$${}^4\log x^2 + {}^4\log y + {}^4\log z = 2$$

$${}^4\log x^2 yz = 2$$

$$x^2 yz = 16 \ldots. (1)$$

$${}^3\log y + {}^9\log z + {}^9\log x = 2$$

$${}^9\log y^2 + {}^9\log z + {}^9\log x = 2$$

$${}^9\log xy^2 z = 2$$

$$xy^2 z = 81 \ldots. (2)$$

$${}^4\log z + {}^{16}\log x + {}^{16}\log y = 2$$

$${}^{16}\log z^2 + {}^{16}\log x + {}^{16}\log y = 2$$

$${}^{16}\log xyz^2 = 2$$

$$xyz^2 = 256 \ldots. (3)$$

Hasil kali ketiga persamaan itu menghasilkan:

$$x^2 yz \times xy^2 z \times xyz^2 = 16 \times 81 \times 256$$

$$x^4y^4z^4 = 2^4 \times 3^4 \times 4^4$$
$$xyz = 2 \times 3 \times 4$$
$$xyz = 24$$
$$xyz = 24 \rightarrow x^2yz = 16$$
$$24x = 16$$
$$x = \frac{2}{3}$$
$$xyz = 24 \rightarrow xy^2z = 81$$
$$24y = 81$$
$$y = \frac{27}{8}$$
$$xyz = 24 \rightarrow xyz^2 = 256$$
$$24z = 256$$
$$z = \frac{32}{3}$$

Jadi, himpunan solusinya adalah $\left\{\left(\frac{2}{3}, \frac{27}{8}, \frac{32}{3}\right)\right\}$.

24. Jika (x, y, z) adalah solusi dari system persamaan berikut ini.

$$\begin{cases} {}^{2}\log x + {}^{4}\log y + {}^{4}\log z = {}^{4}\log 16 \\ {}^{3}\log y + {}^{9}\log x + {}^{9}\log z = {}^{9}\log 81 \\ {}^{4}\log z + {}^{16}\log x + {}^{16}\log y = {}^{16}\log 256 \end{cases}$$. Carilah nilai dari $\frac{48192}{xyz}$.

Solusi:

Dafinisi: ${}^{a}\log x = y \Leftrightarrow x = a^y$

Akibat 1: ${}^{a}\log a^y = y$

Akibat 2: $a^{{}^{a}\log x} = x$

Ketentuan 1: ${}^{a^k}\log x = \frac{1}{k}\,{}^{a}\log x$

Bukti:

$${}^{a^k}\log x = {}^{a^k}\log a^{{}^{a}\log x} = {}^{a^k}\log\left(a^k\right)^{\frac{1}{k}{}^{a}\log x} = \frac{1}{k}\,{}^{a}\log x$$

Ketentuan 2: ${}^{a}\log x^k = k\,{}^{a}\log x$

Bukti:

$$^{a}\log x^{k} = {}^{a}\log\left(a^{{}^{a}\log x}\right)^{k} = {}^{a}\log a^{k\,{}^{a}\log x} = k\,{}^{a}\log x$$

Akibat 3: $^{a^q}\log x^{p} = \frac{p}{q}\,{}^{a}\log x$

Dalam logaritma didefinikan $x > 0, y > 0, z > 0$. Pada basis ini untuk $a > 0, a \neq 1$,

Gunakan $^{a^2}\log A = \frac{{}^{a}\log A}{{}^{a}\log a^2} = \frac{1}{2}\,{}^{a}\log A$ untuk menuliskan kembali sistem persamaan, kita mendapatkan

$$^{2}\log x + {}^{4}\log y + {}^{4}\log z = {}^{4}\log 16 \Leftrightarrow {}^{2}\log x + \frac{1}{2}\,{}^{2}\log y + \frac{1}{2}\,{}^{2}\log z = 2 \Leftrightarrow$$

$$2\,{}^{2}\log x + {}^{2}\log y + {}^{2}\log z = 4 \Leftrightarrow {}^{2}\log x^2yz = 4 \Leftrightarrow x^2yz = 2^4 \Leftrightarrow x^2yz = 4^2 \text{ …. (1)}$$

$$^{3}\log y + {}^{9}\log x + {}^{9}\log z = {}^{9}\log 81 \Leftrightarrow {}^{3}\log y + \frac{1}{2}\,{}^{3}\log x + \frac{1}{2}\,{}^{3}\log z = 2 \Leftrightarrow$$

$$2\,{}^{3}\log y + {}^{3}\log x + {}^{3}\log z = 4 \Leftrightarrow {}^{3}\log y^2xz = 4 \Leftrightarrow y^2xz = 3^4 \Leftrightarrow y^2xz = 9^2 \text{ …. (2)}$$

$$^{4}\log z + {}^{16}\log x + {}^{16}\log y = {}^{16}\log 256 \Leftrightarrow {}^{4}\log z + \frac{1}{2}\,{}^{4}\log x + \frac{1}{2}\,{}^{4}\log y = 2 \Leftrightarrow$$

$$2\,{}^{4}\log z + {}^{4}\log x + {}^{4}\log y = 4 \Leftrightarrow {}^{4}\log z^2xy = 4 \Leftrightarrow z^2xy = 4^4$$

$\Leftrightarrow z^2xy = 16^2$ …. (3)

Kalikan kedua sisi dari persamaan-persamaan ini memberikan

$(xyz)^4 = (4\cdot 9\cdot 16)^2 = (2\cdot 3\cdot 4)^4$, dengan $x > 0, y > 0, z > 0$, menghasilkan $xyz = 2\cdot 3\cdot 4$ …. (4).

Selanjutnya dari (1), (4), kita mendapatkan $(2\cdot 3\cdot 4)x = 4^2 \Leftrightarrow x = \frac{2}{3}$, analogi dari (2), (4) dan (3), (4), kita memperoleh jawaban $x = \frac{2}{3}, y = \frac{27}{8}, z = \frac{32}{3}$.

$$xyz = \frac{2}{3}\times\frac{27}{8}\times\frac{32}{3} = 24$$

$$\therefore \frac{48192}{xyz} = \frac{48192}{24} = 2008$$

25. Tentukan nilai x yang merupakan akar-akar persamaan $6^{-3+\frac{5}{2}\,{}^{6}\log x} = x^{3-\frac{1}{2}\,{}^{6}\log x}$.

Solusi:

$$6^{-3+\frac{5}{2}\,{}^{6}\log x} = x^{3-\frac{1}{2}\,{}^{6}\log x}$$

$$6^{-3+\frac{5}{2}\,{}^{6}\log x} = x^{3-\frac{1}{2}\,{}^{6}\log x}$$

$${}^{6}\log 6^{-3+\frac{5}{2}{}^{6}\log x} = {}^{6}\log x^{3-\frac{1}{2}{}^{6}\log x}$$

$${}^{6}\log 6^{-3} + {}^{6}\log 6^{\frac{5}{2}{}^{6}\log x} = \left(3-\frac{1}{2}{}^{6}\log x\right){}^{6}\log x$$

$$-3+\frac{5}{2}{}^{6}\log x = 3\,{}^{6}\log x - \frac{1}{2}{}^{6}\log^{2} x$$

Misalnya ${}^{6}\log x = y$, sehingga

$$-3+\frac{5}{2}y = 3y - \frac{1}{2}y^{2}$$

$$-6+5y = 6y - y^{2}$$

$$y^{2} - y - 6 = 0$$

$$(y-3)(y+2) = 0$$

$y = 3$ atau $y = -2$

${}^{6}\log x = 3$ atau ${}^{6}\log x = -2$

$x = 216$ atau $x = \frac{1}{36}$

26. Jika akar-akar persamaan $(p+1)x^{3\log x^{2}} - p\cdot\frac{x^{3\log x}}{x^{3}} = \frac{1}{x^{6}}$ adalah a dan b dengan $a > b$, maka nilai $\frac{a}{b} = ...$

Solusi:

$$(p+1)x^{3\log x^{2}} - p\cdot\frac{x^{3\log x}}{x^{3}} = \frac{1}{x^{6}}$$

$$(p+1)x^{6+6\log x} - px^{3+3\log x} = 1$$

Misalnya $x^{3+3\log x} = y$, sehingga

$$(p+1)y^{2} - py - 1 = 0$$

$$(y-1)\left[(p+1)y+1\right] = 0$$

$y = 1$(diterima) atau $y = -\frac{1}{p+1}$(ditolak)

$$x^{3+3\log x} = 1$$

$x = 1$ atau $3+3\log x = 0$

$x = 1$ atau $3+3\log x = 0 \Leftrightarrow x = 10^{-1}$

Sehingga $a = 1$ dan $b = 10^{-1}$

Jadi, $\frac{a}{b}=\frac{1}{10^{-1}}=10$

27. Tentukan himpunan penyelesaian dari sistem persamaan $\begin{cases} {}^{x+y}\log(x-y)+{}^{x-y}\log(x+y)=\frac{34}{15} \\ {}^{a}\log x-{}^{a}\log y={}^{a}\log 5-{}^{a}\log 3 \end{cases}$

Solusi:

$${}^{x+y}\log(x-y)+{}^{x-y}\log(x+y)=\frac{34}{15}$$

$${}^{x+y}\log(x-y)+\frac{1}{{}^{x+y}\log(x-y)}=\frac{34}{15}$$

Misalnya $p={}^{x+y}\log(x-y)$, sehingga

$$p+\frac{1}{p}=\frac{34}{15}$$

$$15p^2-34p+15=0$$

$$(5p-3)(3p-5)=0$$

$$p=\frac{3}{5}\vee p=\frac{5}{3}$$

$${}^{x+y}\log(x-y)=\frac{3}{5}....(1)$$

$${}^{x+y}\log(x-y)=\frac{5}{3}....(1)$$

$${}^{a}\log x-{}^{a}\log y={}^{a}\log 5-{}^{a}\log 3$$

$${}^{a}\log\frac{x}{y}={}^{a}\log\frac{5}{3}$$

$$\frac{x}{y}=\frac{5}{3}$$

$$y=\frac{3x}{5} \text{ .... (3)}$$

Dari (1) dan (3) diperoleh

$${}^{x+\frac{3}{5}x}\log\left(x-\frac{3}{5}x\right)=\frac{3}{5}$$

$${}^{\frac{8}{5}x}\log\frac{2}{5}x=\frac{3}{5}$$

$$\frac{\log \frac{2}{5}x}{\log \frac{8}{5}x} = \frac{3}{5}$$

$$\frac{\log 2x - \log 5}{\log 8x - \log 5} = \frac{3}{5}$$

$$5\log 2x - 5\log 5 = 3\log 8x - 3\log 5$$

$$\log(8x)^3 - \log(2x)^5 = 3\log 5 - 5\log 5$$

$$\log \frac{2^9 x^3}{2^5 x^5} = -2\log 5$$

$$\log \frac{16}{x^2} = \log \frac{1}{25}$$

$$\frac{16}{x^2} = \frac{1}{25}$$

$$x^2 = 16 \cdot 25$$

$$x = \pm\sqrt{16 \cdot 25} = \pm 20$$

$$x = 20 \rightarrow y = \frac{3 \cdot 20}{5} = 12 \text{ (diterima)}$$

$$x = -20 \rightarrow y = \frac{3(-20)}{5} = -12 \text{ (ditolak)}$$

Dari (2) dan (3) diperoleh

$${}^{x+\frac{3}{5}x}\log\left(x - \frac{3}{5}x\right) = \frac{5}{3}$$

$${}^{\frac{8}{5}x}\log \frac{2}{5}x = \frac{5}{3}$$

$$\frac{\log \frac{2}{5}x}{\log \frac{8}{5}x} = \frac{5}{3}$$

$$\frac{\log 2x - \log 5}{\log 8x - \log 5} = \frac{5}{3}$$

$$3\log 2x - 3\log 5 = 5\log 8x - 5\log 5$$

$$\log(8x)^5 - \log(2x)^3 = 5\log 5 - 3\log 5$$

$$\log \frac{2^{15} x^5}{2^3 x^3} = 2\log 5$$

$$\log 2^{12} x^2 = \log 25$$

$$2^{12}x^2 = 25$$

$$x^2 = \frac{25}{2^{12}}$$

$$x = \pm\sqrt{\frac{25}{2^{12}}} = \pm\frac{5}{2^6} = \pm\frac{5}{64}$$

$$x = \frac{5}{64} \to y = \frac{3\cdot\frac{5}{64}}{5} = \frac{3}{64} \text{ (diterima)}$$

$$x = -\frac{5}{64} \to y = \frac{3\left(-\frac{5}{64}\right)}{5} = -\frac{3}{64} \text{ (ditolak)}$$

Jadi, himpunan penyelesaiaannya adalah $\left\{(20,12),\left(\frac{5}{64},\frac{3}{64}\right)\right\}$

28. Tentukan himpunan penyelesaian dari sistem persamaan
$$\begin{cases} {}^{x}\log y + 6\,{}^{y}\log x = 5 \\ 2\log x \cdot \log y = \log xy \end{cases}$$

Solusi:

$$2\log x \cdot \log y = \log xy \text{ .... (1)}$$

$${}^{x}\log y + 6\,{}^{y}\log x = 5$$

$${}^{x}\log y + \frac{6}{{}^{x}\log y} = 5$$

Misalnya ${}^{x}\log y = a$, sehingga

$$a + \frac{6}{a} = 5$$

$$a^2 - 5a + 6 = 0$$

$$(a-2)(a-3) = 0$$

$$a = 2 \vee a = 3$$

$${}^{x}\log y = 2 \Leftrightarrow y = x^2 \text{....(2)}$$

$${}^{x}\log y = 3 \Leftrightarrow y = x^3 \text{....(3)}$$

Dari persamaan (1) dan (2) diperoleh

$$2\log x \cdot \log x^2 = \log x \cdot x^2$$

$$4\log^2 x = 3\log x$$

$$\log x(4\log x - 3) = 0$$

$$\log x = 0 \Leftrightarrow x = 1 \text{(ditolak)} \vee \log x = \frac{3}{4} \Leftrightarrow x = 10^{\frac{3}{4}} = \sqrt[4]{1000} \text{(diterima)}$$

$$x = 10^{\frac{3}{4}} \to y = \left(10^{\frac{3}{4}}\right)^2 = 10^{\frac{3}{2}} = \sqrt{1000}$$

Dari persamaan (1) dan (3) diperoleh

$$2\log x \cdot \log x^3 = \log x \cdot x^3$$

$$6\log^2 x = 4\log x$$

$$2\log x(3\log x - 2) = 0$$

$$\log x = 0 \Leftrightarrow x = 1 (\text{ditolak}) \vee \log x = \frac{2}{3} \Leftrightarrow x = 10^{\frac{2}{3}} = \sqrt[3]{100} (\text{diterima})$$

$$x = 10^{\frac{2}{3}} \to y = \left(10^{\frac{2}{3}}\right)^3 = 10^2 = 100$$

Jadi, himpunan penyelesaiaannya adalah $\left\{\left(\sqrt[4]{1000}, \sqrt{1000}\right), \left(\sqrt[3]{100}, 100\right)\right\}$

29. Tentukan nilai x dari persamaan $2x^{{}^6\log x} + 42x^{{}^{\frac{1}{6}}\log x} = 19$.

Solusi:

$$2x^{{}^6\log x} + 42x^{{}^{\frac{1}{6}}\log x} = 19$$

$$2x^{{}^6\log x} + 42x^{-{}^6\log x} = 19$$

Misalnya $y = x^{{}^6\log x}$, sehingga

$$2y + 42y^{-1} = 19$$

$$2y^2 + 42 = 19y$$

$$2y^2 - 19y + 42 = 0$$

$$(2y - 7)(y - 6) = 0$$

$$y = \frac{7}{2} \vee y = 6$$

$$x^{{}^6\log x} = \frac{7}{2} \vee x^{{}^6\log x} = 6$$

$${}^6\log x^{{}^6\log x} = {}^6\log\frac{7}{2} \vee {}^6\log x^{{}^6\log x} = {}^6\log 6$$

$${}^6\log^2 x = {}^6\log\frac{7}{2} \vee {}^6\log^2 x = 1$$

$${}^6\log x = \pm\sqrt{{}^6\log\frac{7}{2}} \vee {}^6\log x = \pm 1$$

$$x = 6^{\pm\sqrt{{}^6\log\frac{7}{2}}} \vee x = 6^{\pm 1}$$

$x_1 = 6^{\sqrt{^6\log\frac{7}{2}}} \vee x_2 = 6^{-\sqrt{^6\log\frac{7}{2}}} \vee x_3 = 6 \vee x_4 = \frac{1}{6}$

30. Tentukan nilai x dari persamaan $^{(x+1)}\log(x-1) - ^{\frac{1}{x+1}}\log 8 = \frac{\log^2 5 - \log^2 2}{\log\sqrt{2,5}}$.

Solusi:

$$^{(x+1)}\log(x-1) - ^{\frac{1}{x+1}}\log 8 = \frac{\log^2 5 - \log^2 2}{\log\sqrt{2,5}}$$

$$^{(x+1)}\log(x-1) - \frac{\log 8}{\log\frac{1}{x+1}} = \frac{(\log 5 + \log 2)(\log 5 - \log 2)}{\log\sqrt{2,5}}$$

$$^{(x+1)}\log(x-1) - \frac{\log 8}{-\log(x-1)} = \frac{(\log 10)(\log 2,5)}{\frac{1}{2}\log 2,5}$$

$$^{(x+1)}\log(x-1) + ^{(x+1)}\log 8 = 2$$

$$^{(x+1)}\log 8(x-1) = 2$$

$$8(x-1) = (x+1)^2$$

$$8x - 8 = x^2 + 2x + 1$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

$$x = 3$$