PENGAYAAN MATEMATIKA
SOLUSI GEOMETRI 2

1. COMC, 1997

Determine all points on the straight line which joins $(-4, 11)$ to $(16, -1)$ and whose coordinates are positive integers.

Solusi:

Persamaan garis mempunyai gradien $m=\frac{-1-11}{16+4}=-\frac{12}{20}=-\frac{3}{5}$

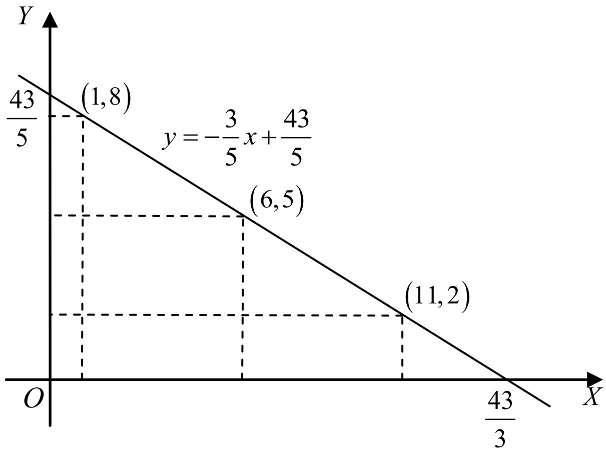
Persamaan garisnya adalah

$y-y_1=m\left(x-x_1\right)$

$y-11=-\frac{3}{5}\left(x+4\right)$

$y=-\frac{3}{5}x-\frac{12}{5}+11$

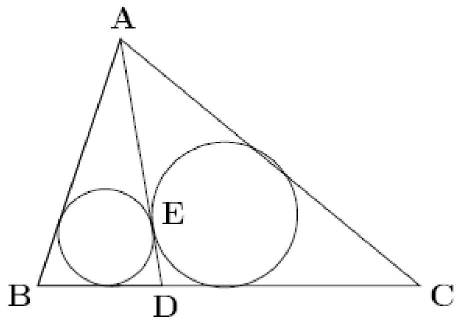
$y=-\frac{3}{5}x+\frac{43}{5}$



Jadi, koordinat-koordinat yang lain yang merupakan pasangan bilangan bulat positif adalah $(1,8);(6,5);\text{dan}(11,2)$.

2. COMC, 1997

The triangle ABC has sides $AB = 137$, $AC = 241$, and $BC = 200$. There is a point D, on BC, such that both incircles of triangles ABD and ACD touch AD at the same point E. Determine the length of CD.



Solusi:

$x+y=241$ .... (1)

$x+w=137$ .... (2)

$y+2z+w=200$ .... (3)

Jumlah ketiga persamaan adalah

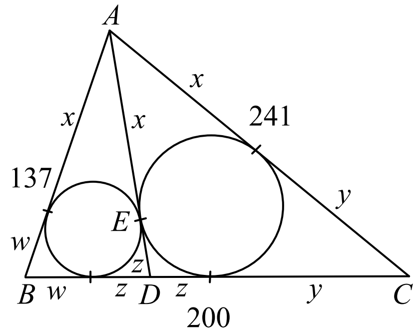
$2x+2y+2z+2w=578$

$x+y+z+w=289$

$y+z+137=289$

$y+z=289-137=152$

$\therefore CD=y+z=152$



3. COMC, 1997

Determine the minimum value of $f(x)$ where

$$f(x) = (3\sin x - 4\cos x - 10)(3\sin x + 4\cos x - 10).$$

Solusi:

$f(x) = (3\sin x - 4\cos x - 10)(3\sin x + 4\cos x - 10) = \left[(3\sin x - 10) - 4\cos x\right]\left[(3\sin x - 10) + 4\cos x\right]$

$= (3\sin x - 10)^2 - (4\cos x)^2 = 9\sin^2 x - 60\sin x + 100 - 16\cos^2 x = 9\sin^2 x - 60\sin x + 100 - 16\left(1 - \sin^2 x\right)$

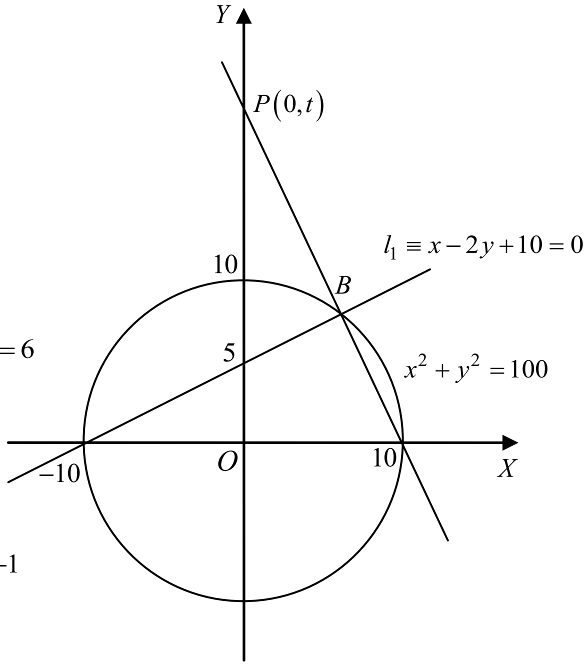
$= 25\sin^2 x - 60\sin x + 84 = (5\sin x - 6)^2 + 48$

Fungsi f mencapai nilai untuk $\sin x \le 1$, sehingga nilai minimum fungsi f adalah $(5\cdot 1 - 6)^2 + 48 = 1 + 48 = 49$.

4. COMC, 1997

The straight line l_1 with equation $x - 2y + 10 = 0$ meets the circle with equation $x^2 + y^2 = 100$ at B in the first quadrant. A line through B, perpendicular to l_1 cuts the y-axis at $P(0, t)$. Determine the value of t.

Solusi:



$x = 2y - 10 \rightarrow x^2 + y^2 = 100$

$(2y - 10)^2 + y^2 = 100$

$4y^2 - 40y + 100 + y^2 = 100$

$5y^2 - 40y = 0$

$5y(y - 8) = 0$

$y = 0$ atau $y = 8$

$x = 2\cdot 0 - 10 = -10$ atau $x = 2\cdot 8 - 10 = 6$

Jadi, koordinat titik $B(6,8)$.

Gradien garis $l_1 \equiv x - 2y + 10 = 0$ adalah

$m_1 = \frac{1}{2}$

Syarat garis saling tegak lurus adalah $m_1 m_2 = -1$

$\frac{1}{2} m_2 = -1$

$m_2 = -2$

Persamaan garis singgung yang melalui titik $B(6,8)$ dan tegak lurus garis $l_1 \equiv x - 2y + 10 = 0$ adalah

$y - y_B = m_2(x - x_B)$

$y - 8 = -2(x - 6)$

$y = -2x + 20$

Garis $y = -2x = 20$ memotong sumbu Y jika $x = 0$, sehingga

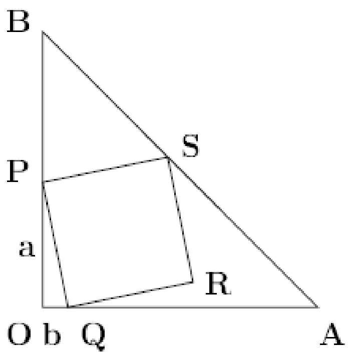
$y = -2\cdot 0 + 20 = 20$

Sehingga koordinat titik $P(0,t) = P(0,20)$.

Jadi, nilai $t = 20$.

5. COMC, 1997

In an isosceles right-angled triangle AOB, points P, Q and S are chosen on sides OB, OA and AB respectively such that a square $PQRS$ is formed as shown. If the lengths of OP and OQ are a and b respectively, and the area of $PQRS$ is $\frac{2}{5}$ that of triangle AOB, determine $a : b$.

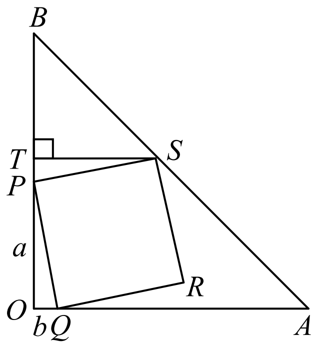


Solusi:

Tarik garis $ST \perp OB$, sehingga $\Delta STP \cong \Delta POQ$, maka $ST = a$ dan $TP = b$.

Karena ΔBTS siku-siku sama kaki, maka $BP = a$.

Jadi, $OB = 2a + b$



$[AOB] = \frac{1}{2}(2a+b)^2$

$PQ = \sqrt{a^2 + b^2}$

$[PQRS] = \left(\sqrt{a^2+b^2}\right)^2 = a^2 + b^2$

$[PQRS] = \frac{2}{5}[AOB]$

$a^2 + b^2 = \frac{2}{5} \times \frac{1}{2}(2a+b)^2$

$5a^2 + 5b^2 = 4a^2 + 4ab + b^2$

$a^2 - 4ab + 4b^2 = 0$

$(a-2b)^2 = 0$

$a = 2b$

$\therefore a : b = 2 : 1$