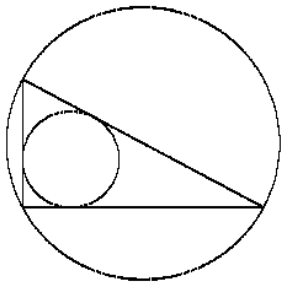
PENGAYAAN MATEMATIKA
SOLUSI GEOMETRI 1

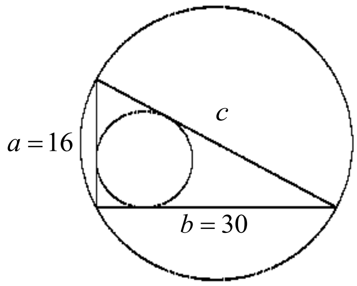
1. COMC, 1996

The vertices of a right angled triangle are on a circle of radius R and the sides of the triangle are tangent to another circle of radius r. If the lengths of the sides about the right angle are 16 and 30, determine the value of $R + r$.



Solusi:

Panjang sisi siku-siku segitiga adalah $a = 16$ dan $b = 30$, sehingga panjang sisi miringnya $c = \sqrt{a^2 + b^2} = \sqrt{16^2 + 30^2} = 34$.



Setengah keliling segitiga ABC adalah

$$s = \frac{1}{2}(a + b + c) = \frac{1}{2}(16 + 30 + 34) = 40$$

Luas segitiga ABC adalah $[ABC] = \frac{1}{2}ab = \frac{1}{2} \times 16 \times 30 = 240$

$$R = \frac{abc}{4[ABC]} = \frac{16 \times 30 \times 34}{4 \times 240} = 17$$

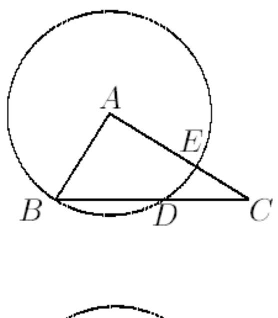
Kita mengetahui juga bahwa jari-jari lingkaran luar (lingkaran terbesar) $R = \frac{1}{2}c = \frac{1}{2} \times 34 = 17$

Jari-jari lingkaran dalam $r = \frac{[ABC]}{s} = \frac{240}{40} = 6$

Jadi, nilai $R + r = 17 + 6 = 23$

2. COMC, 1996

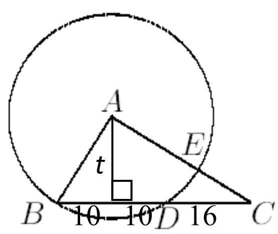
Triangle ABC is right angled at A. The circle with center A and radius AB cuts BC and AC internally at D and E respectively. If $BD = 20$ and $DC = 16$, determine AC^2.



Solusi:

$t^2 = 10 \times 26 = 260$

$AC^2 = t2 + 26^2 = 260 + 676 = 936$



3. COMC, 1996

Determine the sum of the angles A, B, where $0° \le A, B \le 180°$ and $\sin A + \sin B = \sqrt{\frac{3}{2}}$, $\cos A + \cos B = \sqrt{\frac{1}{2}}$.

Solusi:

Kuadratkan kedua persamaan tersebut, sehingga diperoleh

$\cos^2 A + \cos^2 B + 2\cos A \cos B = \frac{3}{2}$ .... (1)

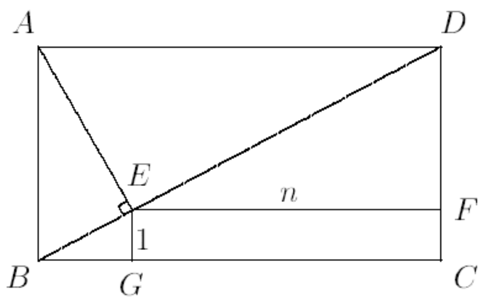
$\sin^2 A + \sin^2 B + 2\sin A \sin B = \frac{1}{2}$ .... (2)

Persamaan (1) + Persamaan (2) menghasilkan

$$2\cos A\cos B + 2\sin A \sin B = 2$$
$$\cos A\cos B + \sin A\sin B = 1$$
$$\cos(A - B) = 1$$
$$A - B = \pm 90°$$
$$\cos A + \cos B = \sqrt{\frac{1}{2}}$$
$$2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B) = \frac{1}{2}\sqrt{2}$$
$$2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(\pm 90°) = \frac{1}{2}\sqrt{2}$$
$$2\cos\frac{1}{2}(A+B)\cos(\pm 45°) = \frac{1}{2}\sqrt{2}$$
$$2\cos\frac{1}{2}(A+B)\times\frac{1}{2}\sqrt{2} = \frac{1}{2}\sqrt{2}$$
$$\cos\frac{1}{2}(A+B) = \frac{1}{2}$$
$$\frac{1}{2}(A+B) = 60°$$
$$A + B = 120°$$

4. COMC, 1996

A rectangle $ABCD$ has diagonal of length d. The line AE is drawn perpendicular to the diagonal BD. The sides of the rectangle $EFCG$ have lengths n and 1. Prove $d^{2/3} = n^{2/3} + 1$.



Solusi:

Misalnya $BG = x$, sehingga $BE = \sqrt{x^2 + 1}$

Perhatikan $\Delta ABE \sim \Delta BEG$, sehingga

$$\frac{AB}{BE} = \frac{BE}{EG}$$

$$\frac{AB}{\sqrt{x^2+1}} = \frac{\sqrt{x^2+1}}{1}$$

$\therefore AB = x^2 + 1$

$x^2 = DF \times 1$

$\therefore DF = x^2$

$AD = n + x$

Perhatikan $\Delta BGE \sim \Delta EFD$, sehingga

$$\frac{BG}{GE} = \frac{EF}{FD}$$

$$\frac{x}{1} = \frac{n}{x^2}$$

$\therefore n = x^3$

$BD^2 = AB^2 + AD^2$

$d^2 = \left(x^2+1\right)^2 + \left(n+x\right)^2$

$d^2 = \left(x^2+1\right)^2 + \left(x^3+x\right)^2$

$d^2 = x^4 + 2x^2 + 1 + x^6 + 2x^4 + x^2$

$d^2 = 3x^4 + 3x^2 + 1 + x^6$

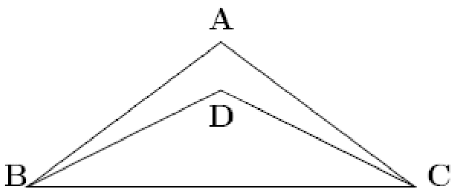
$d^2 = \left(x^2+1\right)^3$

$d^{\frac{2}{3}} = x^2 + 1$

$d^{\frac{2}{3}} = n^{\frac{2}{3}} + 1$ (QED)

5. COMC, 1997

In triangle ABC, $\angle A$ equals 120 degrees. A point D is inside the triangle such that $\angle DBC = 2 \cdot \angle ABD$ and $\angle DCB = 2 \cdot \angle ACD$. Determine the measure, in degrees, of $\angle BDC$.



Solusi:

Misalnya $\angle ABD = x$ dan $\angle ACD = y$, sehingga $\angle DBC = 2x$ dan $\angle DCB = 2y$.

$\angle ABC + \angle ACB + \angle BAC = 3x + 3y + 120° = 180°$

$\therefore x + y = 20°$

$\angle BDC = 180° - \left(2x + 2y\right) = 180° - 2\left(20°\right) = 140°$