

Solusi Pengayaan Matematika

Edisi 1

Nomor Soal: 1-10

1. Bentuk sederhana dari $7\left(\frac{35}{6}\right)^n : \frac{5^n \cdot 7^{n+1} + 7^{n+2} \cdot 25^n}{6^n + 7 \cdot 30^n}$ dengan n bilangan bulat adalah
- A. 7 B. 5 C. 4 D. 3 E. 1

Solusi: [E]

$$\begin{aligned} 7\left(\frac{35}{6}\right)^n : \frac{5^n \cdot 7^{n+1} + 7^{n+2} \cdot 25^n}{6^n + 7 \cdot 30^n} &= \frac{7 \cdot 7^n \cdot 5^n}{6^n} \times \frac{6^n + 7 \cdot 6^n \cdot 5^n}{5^n \cdot 7^{n+1} + 7^{n+2} \cdot 5^{2n}} \\ &= \frac{7 \cdot 7^n \cdot 5^n}{6^n} \times \frac{6^n(1 + 7 \cdot 5^n)}{7^{n+1} \cdot 5^n(1 + 7 \cdot 5^n)} = 1 \end{aligned}$$

2. Bentuk sederhana dari $\sqrt{3+\sqrt{8}} - \sqrt{3-\sqrt{8}}$ adalah
- A. 2 B. 3 C. 5 D. 8 E. 11

Solusi 1: [A]

Karena $\sqrt{3+\sqrt{8}} > 0$, $\sqrt{3-\sqrt{8}} > 0$, dan $\sqrt{3+\sqrt{8}} > \sqrt{3-\sqrt{8}}$ maka $\sqrt{3+\sqrt{8}} - \sqrt{3-\sqrt{8}} > 0$.

$$\begin{aligned} \left(\sqrt{3+\sqrt{8}} - \sqrt{3-\sqrt{8}}\right)^2 &= 3 + \sqrt{8} - 2\sqrt{(3+\sqrt{8})(3-\sqrt{8})} + 3 - \sqrt{8} = 6 - 2\sqrt{9-8} \\ &= 6 - 2 \times 1 = 4 \\ \therefore \sqrt{3+\sqrt{8}} - \sqrt{3-\sqrt{8}} &= \sqrt{4} = 2 \end{aligned}$$

Solusi 2: [A]

$$\begin{aligned} \sqrt{3+\sqrt{8}} - \sqrt{3-\sqrt{8}} &= \sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}} = \sqrt{2} + \sqrt{1} - (\sqrt{2} - \sqrt{1}) \\ &= \sqrt{2} + 1 - \sqrt{2} + 1 = 2 \end{aligned}$$

3. Bentuk sederhana dari $\sqrt{13+30\sqrt{2+\sqrt{9+4\sqrt{2}}}}$ adalah
- A. $5+3\sqrt{2}$ B. $3+\sqrt{2}$ C. $5-\sqrt{2}$ D. $5+2\sqrt{3}$ E. $3+5\sqrt{2}$

Solusi: [A]

$$\begin{aligned} \sqrt{13+30\sqrt{2+\sqrt{9+4\sqrt{2}}}} &= \sqrt{13+30\sqrt{2+\sqrt{9+2\sqrt{8}}}} = \sqrt{13+30\sqrt{2+2\sqrt{2}+1}} \\ &= \sqrt{13+30\sqrt{3+2\sqrt{2}}} = \sqrt{13+30(\sqrt{2}+1)} \end{aligned}$$

$$= \sqrt{43 + 30\sqrt{2}} = \sqrt{43 + 2\sqrt{450}} = \sqrt{25} + \sqrt{18}$$

$$= 5 + 3\sqrt{2}$$

4. Bentuk sederhana dari $\frac{\sqrt{3+\sqrt{5}} + \sqrt{4+\sqrt{7}}}{\sqrt{7} + \sqrt{5} + 2} = \dots$

A. $\frac{1}{2}\sqrt{2}$ B. $\frac{1}{2}\sqrt{3}$ C. $2\sqrt{2}$ D. $2\sqrt{3}$ E. $3\sqrt{2}$

Solusi: [A]

$$\frac{\sqrt{3+\sqrt{5}} + \sqrt{4+\sqrt{7}}}{\sqrt{7} + \sqrt{5} + 2} = \frac{\sqrt{\frac{12+4\sqrt{5}}{4}} + \sqrt{\frac{16+4\sqrt{7}}{4}}}{\sqrt{7} + \sqrt{5} + 2}$$

$$= \frac{\frac{1}{2}\sqrt{12+2\sqrt{20}} + \frac{1}{2}\sqrt{16+2\sqrt{28}}}{\sqrt{7} + \sqrt{5} + 2}$$

$$= \frac{\frac{1}{2}\sqrt{10} + \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{14} + \frac{1}{2}\sqrt{2}}{\sqrt{7} + \sqrt{5} + 2} = \frac{1}{2} \times \frac{\sqrt{14} + \sqrt{10} + 2\sqrt{2}}{\sqrt{7} + \sqrt{5} + 2}$$

$$= \frac{1}{2} \times \frac{\sqrt{2}(\sqrt{7} + \sqrt{5} + 2)}{\sqrt{7} + \sqrt{5} + 2} = \frac{1}{2}\sqrt{2}$$

5. Bentuk sederhana dari $\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} + \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}$ adalah

A. 1 B. 2 C. 4 D. 6 E. 8

Solusi: [C]

$$\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} + \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}} = \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}} + \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}}$$

$$= \sqrt{\frac{7+4\sqrt{3}}{4-3}} + \sqrt{\frac{7-4\sqrt{3}}{4-3}} = \sqrt{7+4\sqrt{3}} + \sqrt{7-4\sqrt{3}}$$

$$= \sqrt{7+2\sqrt{12}} + \sqrt{7-2\sqrt{12}} = \sqrt{4} + \sqrt{3} + \sqrt{4} - \sqrt{3} = 4$$

6. Jumlah angka dari hasil kali

$$(\sqrt{5} + \sqrt{6} + \sqrt{7})(-\sqrt{5} + \sqrt{6} + \sqrt{7})(\sqrt{5} - \sqrt{6} + \sqrt{7})(\sqrt{5} + \sqrt{6} - \sqrt{7}) \text{ adalah ...}$$

A. 15 B. 10 C. 9 D. 7 E. 5

Solusi: [D]

$$(\sqrt{5} + \sqrt{6} + \sqrt{7})(-\sqrt{5} + \sqrt{6} + \sqrt{7})(\sqrt{5} - \sqrt{6} + \sqrt{7})(\sqrt{5} + \sqrt{6} - \sqrt{7})$$

$$= [(\sqrt{6} + \sqrt{7}) + \sqrt{5}][(\sqrt{6} + \sqrt{7}) - \sqrt{5}][\sqrt{5} - (\sqrt{6} - \sqrt{7})][\sqrt{5} + (\sqrt{6} - \sqrt{7})]$$

$$\begin{aligned}
&= \left[(\sqrt{6} + \sqrt{7})^2 - (\sqrt{5})^2 \right] \left[(\sqrt{5})^2 - (\sqrt{6} - \sqrt{7})^2 \right] \\
&= (13 + 2\sqrt{42} - 5)(5 - 13 + 2\sqrt{42}) = (8 + 2\sqrt{42})(-8 + 2\sqrt{42}) \\
&= (2\sqrt{42})^2 - (8)^2 = 168 - 64 = 104
\end{aligned}$$

Jadi, jumlah angka-angkanya adalah $1 + 0 + 4 = 5$.

7. Jika $x = \sqrt{2 + \sqrt{2}} - \sqrt{2 - \sqrt{2}}$, maka nilai dari $384x^2 - x^8$ adalah
 A. 844 B. 484 C. 488 D. 448 E. 444

Solusi: [D]

$$\begin{aligned}
x^2 &= 2 + \sqrt{2} + 2 - \sqrt{2} + 2\sqrt{2} = 4 - 2\sqrt{2} \\
x^6 &= (x^2)^3 = 64 - 3(16)(2\sqrt{2}) + 3(4)(8) - 16\sqrt{2} = 160 - 112\sqrt{2} \\
\therefore 384x^2 - x^8 &= x^2(384 - x^6) = (4 - 2\sqrt{2})(384 - 160 + 112\sqrt{2}) \\
&= (4 - 2\sqrt{2})(224 + 112\sqrt{2}) = 224(2 - \sqrt{2})(2 + \sqrt{2}) = 224(4 - 2) = 448
\end{aligned}$$

8. Bentuk sederhana dari $\sqrt[3]{2 + \frac{10}{9}\sqrt{3}} + \sqrt[3]{2 - \frac{10}{9}\sqrt{3}}$ adalah
 A. 1 B. 2 C. 3 D. 4 E. 5

Solusi: [B]

Kita mengetahui bahwa $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$.

Misalnya $x = \sqrt[3]{2 + \frac{10}{9}\sqrt{3}} + \sqrt[3]{2 - \frac{10}{9}\sqrt{3}}$ dan pangkatkan kedua ruasnya, sehingga diperoleh

$$x^3 = 2 + \frac{10}{9}\sqrt{3} + 2 - \frac{10}{9}\sqrt{3} + 3\sqrt{\left(2 + \frac{10}{9}\sqrt{3}\right)\left(2 - \frac{10}{9}\sqrt{3}\right)} \left(\sqrt[3]{2 + \frac{10}{9}\sqrt{3}} + \sqrt[3]{2 - \frac{10}{9}\sqrt{3}}\right)$$

$$x^3 = 4 + 3\sqrt[3]{4 - \frac{100}{27}} \cdot x$$

$$x^3 = 4 + 3x\sqrt[3]{\frac{8}{27}} = 4 + 2x$$

$$x^3 - 2x - 4 = 0$$

$$(x - 2)(x^2 + 2x + 2) = 0$$

$$x = 2 \text{ atau } x^2 + 2x + 2 = 0 \text{ (memberikan akar-akar tidak real)}$$

$$\text{Karena } x > 0, \text{ maka } \sqrt[3]{2 + \frac{10}{9}\sqrt{3}} + \sqrt[3]{2 - \frac{10}{9}\sqrt{3}} = 2.$$

9. Jika pecahan $\frac{1}{1 + \sqrt[3]{2} + 2\sqrt[3]{4}}$ setelah dirasionalkan penyebutnya berbentuk $a + b\sqrt[3]{2} + c\sqrt[3]{4}$, maka nilai dari $23(a + b + c) = \dots$

- A. -3 B. -1 C. 3 D. 4 E. 11

Solusi: [C]

Misalnya $\frac{1}{1+\sqrt[3]{2}+2\sqrt[3]{4}} = a+b\sqrt[3]{2}+c\sqrt[3]{4}$, sehingga

$$(1+\sqrt[3]{2}+2\sqrt[3]{4})(a+b\sqrt[3]{2}+c\sqrt[3]{4})=1$$

$$a+b\sqrt[3]{2}+c\sqrt[3]{4}+a\sqrt[3]{2}+b\sqrt[3]{4}+2c+2a\sqrt[3]{4}+4b+4c\sqrt[3]{2}=1$$

$$a+4b+2c+(a+b+4c)\sqrt[3]{2}+(2a+b+c)c\sqrt[3]{4}=1$$

$$a+4b+2c=1 \dots (1)$$

$$a+b+4c=0 \dots (2)$$

$$2a+b+c=0 \dots (3)$$

$$\text{Persamaan (1) - Persamaan (2): } 3b-2c=1 \dots (4)$$

$$\text{Persamaan (3) - } 2 \times \text{Persamaan (1): } -7b-3c=-2 \dots (5)$$

$$3 \times \text{Persamaan (4) - } 2 \times \text{Persamaan (5): } 23b=7 \Leftrightarrow b=\frac{7}{23}$$

$$\text{Substitusikan } b=\frac{7}{23} \text{ ke persamaan (4), sehingga } 3 \times \frac{7}{23}-2c=1 \Leftrightarrow c=\frac{-1}{23}$$

$$\text{Substitusikan } b=\frac{7}{23} \text{ dan } c=\frac{-1}{23} \text{ ke persamaan (2), sehingga}$$

$$a+\frac{7}{23}+4\left(\frac{-1}{23}\right)=0 \Leftrightarrow a=\frac{-3}{23}$$

$$\text{Jadi, } 23(a+b+c)=23\left(\frac{-3}{23}+\frac{7}{23}+\frac{-1}{23}\right)=3.$$

10. Diketahui $a, b, c, d, e,$ dan f adalah bilangan real. Jika $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=64$, maka

$$\text{nilai dari } \sqrt{\frac{5a^2c-4c^2e+e^3}{5b^2d-4d^2f+f^3}}=\dots$$

- A. 128 B. 192 C. 256 D. 512 E. 1024

Solusi: [D]

Karena $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=64$, maka $a=64b, c=64d$, dan $e=64f$

$$\begin{aligned} \sqrt{\frac{5a^2c-4c^2e+e^3}{5b^2d-4d^2f+f^3}} &= \sqrt{\frac{5(64b)^2(64d)-4(64d)^2(64f)+(64f)^3}{5b^2d-4d^2f+f^3}} \\ &= \sqrt{\frac{64^3(5b^2d-4d^2f+f^3)}{5b^2d-4d^2f+f^3}} = \sqrt{64^3} = \sqrt{8^6} = 8^3 = 512 \end{aligned}$$