Solusi Pengayaan Matematika
Edisi 10

Nomor Soal: 91-100

91. Hitunglah $\sin(947° + 2008°)\sin(1663° - 2008°)$.

A. 2008 B. 1004 C. 502 D. −502 E. −1004

Solusi:[C]

$$-2008\sin(947° + 2008°)\sin(1663° - 2008°) = 2008\sin 2955° \sin -345°$$
$$= 2008\sin(75° + 8\cdot 360°)\sin(360° - 15°) = 2008\sin 75° \sin(-15°)$$
$$= -1004(2\sin 75° \sin 15°) = -1004(\cos 90° - \cos 60°) = -1004\left(0 - \frac{1}{2}\right) = 502$$

92. Dalam ΔABC, sudut B sama dengan dua kali sudut A. Jika sisi b adalah 1,5 kali sisi a, tentukan $\cos 2A$

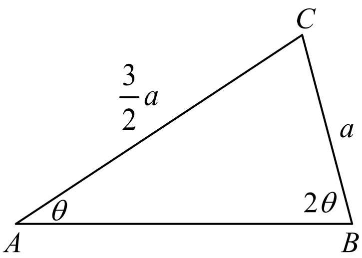
A. $\frac{\sqrt{3}}{4}$ B. $\frac{2}{3}$ C. $\frac{3}{4}$ D. $\frac{1}{8}$ E. $\frac{5}{8}$

Solusi: [D]

Misalnya $\angle A = \theta$, sehingga $\angle B = 2\angle A = 2\theta$ dan $b = 2a$

Menurut Aturan Sinus:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$



$$\frac{a}{\sin\theta} = \frac{\frac{3}{2}a}{\sin 2\theta}$$
$$\frac{\sin 2\theta}{\sin\theta} = \frac{\frac{3}{2}a}{a}$$
$$\frac{2\sin\theta\cos\theta}{\sin\theta} = \frac{3}{2}$$
$$\cos\theta = \frac{3}{4}$$
$$\therefore \cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{3}{4}\right)^2 - 1 = \frac{18}{16} - 1 = \frac{2}{16} = \frac{1}{8}$$

93. Yang mana dari bilangan identitas berikut yang benar?

(1) $(\csc x - \cot x)(1 + \cos x) = \sin x$

(2) $\sec x \csc x - \cot x = \tan x$

(3) $\frac{1 + \cos 2x}{\sin 2x \cos x} = \csc x$

(4) $\cos 2x = 1 - 2\sin^2 x$

A. hanya (1) C. hanya (2) dan (3) E. semua identitas

B. hanya (1) dan (2) D. hanya (1) dan (4)

Solusi: [E]

(1) $(\csc x - \cot x)(1+\cos x) = \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)(1+\cos x) = \frac{1}{\sin x} + \frac{\cos x}{\sin x} - \frac{\cos x}{\sin x} - \frac{\cos^2 x}{\sin x}$

$$= \frac{1}{\sin x}\left(1-\cos^2 x\right) = \frac{1}{\sin x}\left(\sin^2 x\right) = \sin x$$

(2) $\sec x \csc x - \cot x = \frac{1}{\cos x}\cdot\frac{1}{\sin x} - \frac{\cos x}{\sin x} = \frac{1}{\sin x}\left(\frac{1}{\cos x} - \cos x\right) = \frac{1}{\sin x}\left(\frac{1-\cos^2 x}{\cos x}\right)$

$$= \frac{1}{\sin x}\left(\frac{\sin^2 x}{\cos x}\right) = \frac{\sin x}{\cos x} = \tan x$$

(3) $\frac{1+\cos 2x}{\sin 2x \cos x} = \frac{1+2\cos^2 x - 1}{2\sin x \cos x \cos x} = \frac{2\cos^2 x}{2\sin x \cos^2 x} = \frac{1}{\sin x} = \csc x$

(4) $\cos(x+x) = \cos x \cos x - \sin x \sin x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

Jadi, semua identitas benar.

94. Tentukan nilai dari $\cos 22{,}5^\circ - \sin 22{,}5^\circ \cot 11{,}25^\circ$.

A. 2 B. 1 C. 0 D. –1 E. –2

Solusi: [D]

$$\cos 22{,}5^\circ - \sin 22{,}5^\circ \cot 11{,}25^\circ = \cos 22{,}5^\circ - \sin 22{,}5^\circ \frac{\cos 11{,}25^\circ}{\sin 11{,}25}$$

$$= \frac{\cos 22{,}5^\circ \sin 11{,}25^\circ - \sin 22{,}5^\circ \cos 11{,}25^\circ}{\sin 11{,}25^\circ}$$

$$= \frac{\sin(11{,}25^\circ - 22{,}5^\circ)}{\sin 11{,}25^\circ} = \frac{\sin(-11{,}25^\circ)}{\sin 11{,}25^\circ} = -1$$

95. Jika $x + \frac{1}{x} = \sqrt{2}$, hitunglah $x^8 + \frac{1}{x^8}$.

A. $8\sqrt{2}$ B. 4 C. $2\sqrt{2}$ D. 2 E. $\sqrt{2}$

Solusi 1: [D]

Misalnya $x + \frac{1}{x} = 2\cos\theta$ sehingga

$x^2 - (2\cos\theta)x + 1 = 0$

Jika $x = \cos\theta \pm i\sin\theta$, maka $x^n = \cos n\theta \pm i\sin n\theta$ dan $x^{-n} = \cos n\theta \mp i\sin n\theta$, sehingga $x^n + \frac{1}{x^n} = 2\cos n\theta$.

Misalnya $\sqrt{2} = 2\cos\frac{\pi}{4}$ dan $n = 8$.

Jadi, $x^8 + \frac{1}{x^8} = 2\cos 8\left(\frac{\pi}{4}\right) = 2$.

Solusi 2: [D]

$x + \frac{1}{x} = \sqrt{2}$

$\left(x + \frac{1}{x}\right)^2 = \left(\sqrt{2}\right)^2$

$x^2 + 2 + \frac{1}{x^2} = 2$

$x^2 + \frac{1}{x^2} = 0$

$\left(x^2 + \frac{1}{x^2}\right)^2 = 0^2$

$x^4 + 2 + \frac{1}{x^4} = 0$

$x^4 + \frac{1}{x^4} = -2$

$\left(x^4 + \frac{1}{x^4}\right)^2 = (-2)^2$

$x^8 + 2 + \frac{1}{x^8} = 4$

$x^8 + \frac{1}{x^8} = 2$

96. Jika $\tan a + \tan b = 24$ dan $\cot a + \cot b = 32$, berapakah $\tan(a+b)$?

A. 288 B. 144 C. 108 D. 96 E. 64

Solusi: [D]

$\cot a + \cot b = 32$

$\frac{1}{\tan a} + \frac{1}{\tan b} = 32$

$\frac{\tan a + \tan b}{\tan a \tan b} = 32$

$\tan a \tan b = \frac{\tan a + \tan b}{32} = \frac{24}{32} = \frac{3}{4}$

$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b} = \frac{24}{1 - \frac{3}{4}} = 96$

97. Persamaan kuadrat (polinomial kuadrat) $ax^2 + bx + c = 0$ dengan koefisien a, b, dan c bulat. Jika akar-akarnya adalah $\cos 72°$ dan $\cos 144°$, tentukan nilai $a + b + c$.

A. 8 B. 7 C. 6 D. 5 E. 4

Solusi: [D]

Karenanya, $\cos 72° = \cos(90° - 18°) = \sin 18° = \frac{\sqrt{5} - 1}{4}$ dan

$\cos 144° = \cos(180° - 36°) = -\cos 36° = -\frac{1 + \sqrt{5}}{4}$

$\therefore \cos 72° + \cos 144° = -\frac{1}{2}$ dan $\cos 72° \cdot \cos 144° = -\frac{1}{4}$

Karenanya persamaan kuadrat yang dinginkan adalah $x^2 + \frac{1}{2}x - \frac{1}{4} = 0 \Leftrightarrow 4x^2 + 2x - 1 = 0$.

Jadi, nilai $a + b + c = 4 + 2 - 1 = 5$

98. Hitunglah jumlah $\left(\cos\frac{\pi}{1000} + \cos\frac{2\pi}{1000} + \cos\frac{3\pi}{1000} + ... + \cos\frac{1999\pi}{1000}\right)^{2008}$

A. 0 B. 1 C. -1 D. $\frac{\sqrt{2}}{2}$ E. $\sqrt{2}$

Solusi: [B]

$\cos(\pi + x) = -\cos x$

$\cos\frac{\pi}{1000} + \cos\frac{2\pi}{1000} + \cos\frac{3\pi}{1000} + ... + \cos\frac{1999\pi}{1000}$

$= \cos\frac{\pi}{1000} + \cos\frac{2\pi}{1000} + ... + \cos\frac{999\pi}{1000} + \cos\frac{1000\pi}{1000} + \cos\frac{1001\pi}{1000} + ... + \cos\frac{1999\pi}{1000}$

$= \cos\frac{\pi}{1000} + \cos\frac{2\pi}{1000} + ... + \cos\frac{999\pi}{1000} + \cos\pi + \cos\left(\pi + \frac{\pi}{1000}\right) + ... + \cos\left(\pi + \frac{999\pi}{1000}\right)$

$= \cos\frac{\pi}{1000} + \cos\frac{2\pi}{1000} + ... + \cos\frac{999\pi}{1000} + \cos\pi - \cos\frac{\pi}{1000} - ... - \cos\frac{999\pi}{1000}$

$= 0 + 0 + ... + 0 + \cos\pi = -1$

$\left(\cos\frac{\pi}{1000} + \cos\frac{2\pi}{1000} + \cos\frac{3\pi}{1000} + ... + \cos\frac{1999\pi}{1000}\right)^{2008} = (-1)^{2008}$

99. Jika $\frac{a - ab^2}{\sqrt{1-b^2}} + \frac{b - a^2b}{\sqrt{1-a^2}} = 1$, dengan $0 \le a, b \le 1$, tentukan nilai $a^2 + b^2$.

A. 5 B. 4 C. 3 D. 2 E. 1

Solusi 1: [E]

Misalnya $a = \sin\alpha$ dan $b = \sin\beta$, sehingga

$\frac{a - ab^2}{\sqrt{1-b^2}} + \frac{b - a^2b}{\sqrt{1-a^2}} = 1$

$\frac{a(1-b^2)}{\sqrt{1-b^2}} \times \frac{\sqrt{1-b^2}}{\sqrt{1-b^2}} + \frac{b(1-a^2)}{\sqrt{1-a^2}} \times \frac{\sqrt{1-a^2}}{\sqrt{1-a^2}} = 1$

$a\sqrt{1-b^2} + b\sqrt{1-a^2} = 1$

$\sin\alpha\sqrt{1-\sin^2\beta} + \sin\beta\sqrt{1-\sin^2\alpha} = 1$

$\sin\alpha\cos\beta + \sin\beta\cos\alpha = 1$

$\sin(\alpha + \beta) = 1$

$\alpha + \beta = 90°$

$a^2 + b^2 = \sin^2\alpha + \sin^2\beta = \sin^2\alpha + \sin^2(90° - \alpha) = \sin^2\alpha + \cos^2\alpha = 1$

Solusi 2: [E]

Misalnya $a = \cos\alpha$ dan $b = \cos\beta$, sehingga

$\frac{a - ab^2}{\sqrt{1-b^2}} + \frac{b - a^2b}{\sqrt{1-a^2}} = 1$

$\frac{a(1-b^2)}{\sqrt{1-b^2}} \times \frac{\sqrt{1-b^2}}{\sqrt{1-b^2}} + \frac{b(1-a^2)}{\sqrt{1-a^2}} \times \frac{\sqrt{1-a^2}}{\sqrt{1-a^2}} = 1$

$a\sqrt{1-b^2} + b\sqrt{1-a^2} = 1$

$\cos\alpha\sqrt{1-\cos^2\beta} + \cos\beta\sqrt{1-\cos^2\alpha} = 1$

$\cos\alpha\sin\beta+\cos\beta\sin\alpha=1$

$\sin(\alpha+\beta)=1$

$\alpha+\beta=90°$

$a^2+b^2=\sin^2\alpha+\sin^2\beta=\sin^2\alpha+\sin^2(90°-\alpha)=\sin^2\alpha+\cos^2\alpha=1$

Solusi 3: [E]

$$\frac{a-ab^2}{\sqrt{1-b^2}}+\frac{b-a^2b}{\sqrt{1-a^2}}=1$$

$$\frac{a(1-b^2)}{\sqrt{1-b^2}}\times\frac{\sqrt{1-b^2}}{\sqrt{1-b^2}}+\frac{b(1-a^2)}{\sqrt{1-a^2}}\times\frac{\sqrt{1-a^2}}{\sqrt{1-a^2}}=1$$

$$a\sqrt{1-b^2}+b\sqrt{1-a^2}=1$$

$$a\sqrt{1-b^2}=-b\sqrt{1-a^2}+1$$

$$a^2(1-b^2)=b^2(1-a^2)-2b\sqrt{1-a^2}+1$$

$$a^2-a^2b^2=b^2-a^2b^2-2b\sqrt{1-a^2}+1$$

$$2b\sqrt{1-a^2}=b^2-a^2+1$$

$$4b^2(1-a^2)=b^4+a^4+1-2a^2b^2+2b^2-2a^2$$

$$4b^2-4a^2b^2=b^4+a^4+1-2a^2b^2+2b^2-2a^2$$

$$a^4+2a^2b^2+b^4-2(a^2+b^2)+1=0$$

$$(a^2+b^2)^2-2(a^2+b^2)+1=0$$

$$(a^2+b^2-1)^2=0$$

$$a^2+b^2-1=0$$

$$a^2+b^2=1$$

100. Diberikan $f(x)=x+\sqrt{3-3x^2}+2$, dengan x adalah bilangan real dan $0\le x\le 1$. Tentukan nilai maksimum f.

A. 4 B. 3 C. 2 D. $\sqrt{3}$ E. 1

Solusi: [A]

Misalnya $x=\cos\theta$, di mana $0\le\theta\le\pi$.

$$f(x)=x+\sqrt{3-3x^2}+2=\cos\theta+\sqrt{3(1-\cos^2\theta)}+2=\cos\theta+\sqrt{3}\sin\theta+2$$

$$f(x)=\cos\theta+\tan\frac{\pi}{3}\sin\theta+2$$

$$f(x)\cos\frac{\pi}{3}=\cos\theta\cos\frac{\pi}{3}+\sin\theta\sin\frac{\pi}{3}+2\cos\frac{\pi}{3}$$

$$f(x)\cos\frac{\pi}{3}=\cos\left(\theta-\frac{\pi}{3}\right)+2\cos\frac{\pi}{3}$$

$$f(x)=2\cos\left(\theta-\frac{\pi}{3}\right)+2$$

Karena nilai kosinus maksimum adalah 1, maka nilai maksimum $f_{\max}=2+2=4$ yang dicapai jika $\theta=\frac{\pi}{3}$, yaitu jika $x=\frac{1}{2}$.