

# SOLUSI SOAL-SOAL LATIHAN MATRIKS NASKAH B

## A. PILIHAN GANDA:

1. Jika  $\begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix} = \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$ , maka nilai dari  $abc = \dots$

- A. 1                      B. 2                      C. 3                      D. 4                      E. 6

**Solusi: [A]**

$$\begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$$

$$g = 0, e = 1, c = 0$$

$$\therefore g + e + c = 0 + 1 + 0 = 1$$

2. Diberikan bahwa  $A = \begin{pmatrix} 2 & -3 \\ 1 & 7 \end{pmatrix}$  dan  $B = \begin{pmatrix} -1 & 4 & 3 \\ 1 & -2 & 5 \end{pmatrix}$ . Jika  $A^2B = \begin{pmatrix} x & 58 & -132 \\ -55 & y & -203 \end{pmatrix}$ , maka nilai

$$x + y = \dots$$

- A. 158                      B. 156                      C. 128                      D. 120                      E. 100

**Solusi:**

$$A^2B = \begin{pmatrix} x & 58 & -132 \\ -55 & y & -203 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -3 \\ 1 & 7 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 7 \end{pmatrix} \begin{pmatrix} -1 & 4 & 3 \\ 1 & -2 & 5 \end{pmatrix} = \begin{pmatrix} x & 58 & -132 \\ -55 & y & -203 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -27 \\ 9 & -46 \end{pmatrix} \begin{pmatrix} -1 & 4 & 3 \\ 1 & -2 & 5 \end{pmatrix} = \begin{pmatrix} x & 58 & -132 \\ -55 & y & -203 \end{pmatrix}$$

$$\begin{pmatrix} -28 & 58 & -132 \\ -55 & 128 & -203 \end{pmatrix} = \begin{pmatrix} x & 58 & -132 \\ -55 & y & -203 \end{pmatrix}$$

$$x = -28, y = 128$$

$$\therefore x + y = -28 + 128 = 100$$

3. Jika  $\begin{pmatrix} 4 & 2 \\ -3 & -1 \end{pmatrix} \begin{pmatrix} p \\ -1 \end{pmatrix} = \begin{pmatrix} 15 \\ q \end{pmatrix}$ , nilai  $p - q = \dots$

- A. 15                      B. 10                      C. 7                      D. 5                      E. 4

**Solusi: [A]**

$$\begin{pmatrix} 4 & 2 \\ -3 & -1 \end{pmatrix} \begin{pmatrix} p \\ -1 \end{pmatrix} = \begin{pmatrix} 15 \\ q \end{pmatrix}$$

$$\begin{pmatrix} 4p - 2 \\ -3p + 1 \end{pmatrix} = \begin{pmatrix} 15 \\ q \end{pmatrix}$$

$$4p - 2 = 15 \Leftrightarrow p = 4$$

$$-3p + 1 = q$$

$$q = -3 \cdot 4 + 1 = -11$$

$$p - q = 4 - (-11) = 15$$

4. Jika  $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 & -3 \\ 3 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 13 \\ 22 \end{pmatrix}$ , maka nilai  $x+10y = \dots$

- A. 25                      B. 35                      C. 40                      D. 45                      E. 54

**Solusi: [E]**

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 & -3 \\ 3 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 13 \\ 22 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{23} \begin{pmatrix} 2 & 3 \\ -3 & 7 \end{pmatrix} \begin{pmatrix} 13 \\ 22 \end{pmatrix} = \frac{1}{23} \begin{pmatrix} 92 \\ 115 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$$\therefore x+10y = 4+10 \cdot 5 = 54$$

5. Jika  $A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$  dan  $BA = \begin{pmatrix} 5 & 5 \\ 4 & 8 \end{pmatrix}$ , maka  $|2B| = \dots$

- A. 4                      B. 8                      C. 12                      D. 16                      E. 32

**Solusi: [D]**

$$BA = \begin{pmatrix} 5 & 5 \\ 4 & 8 \end{pmatrix}$$

$$B = \begin{pmatrix} 5 & 5 \\ 4 & 8 \end{pmatrix} A^{-1} = \begin{pmatrix} 5 & 5 \\ 4 & 8 \end{pmatrix} \frac{1}{5} \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 20 & -5 \\ 20 & 0 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 4 & 0 \end{pmatrix}$$

$$2B = \begin{pmatrix} 8 & -2 \\ 8 & 0 \end{pmatrix}$$

$$|B| = 0+16=16$$

6. Diberikan matriks-matriks  $A = \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}$ ,  $B = \begin{pmatrix} q & p \\ -7 & 0 \end{pmatrix}$ ,  $C = \begin{pmatrix} 0 & 6 \\ -2 & 3r \end{pmatrix}$ . Jika  $AB^T = C^T + \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$ , maka nilai  $p-q$  adalah ...

- A. 4                      B. 3                      C. 2                      D. -2                      E. -3

**Solusi: [A]**

$$AB^T = C^T + \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} q & -7 \\ p & 0 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 6 & 3r \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} p & 0 \\ -2q & 14 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 6 & 3r-1 \end{pmatrix}$$

$$p=1, q=-3$$

$$p+q=1-(-3)=4$$

7. Diberikan titik-titik  $A(5,6)$ ,  $B(4,-3)$ , dan  $C(-9,2)$ . Luas jajar genjang  $ABCD$  adalah ....

- A. 61                      B. 120                      C. 122                      D. 161                      E. 244

**Solusi: [C]**

$$[ABCD] = 2 \times \frac{1}{2} \begin{vmatrix} 5 & 4 & -9 & 5 \\ 6 & -3 & 2 & 6 \end{vmatrix} = |-15+8-54-(24+27+10)| = |-122| = 122$$

8. Diberikan matriks-matriks  $A = \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}$  dan  $B = \begin{pmatrix} 1 & 2 \\ 4 & -1 \end{pmatrix}$ . Jika  $AQ = 2B + A$ , maka matriks

$$Q^{-1} = \dots$$

- A.  $-\frac{1}{9} \begin{pmatrix} 1 & 2 \\ 2 & -5 \end{pmatrix}$       B.  $\frac{1}{9} \begin{pmatrix} -1 & -2 \\ -2 & 5 \end{pmatrix}$       C.  $\frac{1}{9} \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$       D.  $\frac{1}{9} \begin{pmatrix} -1 & 2 \\ 2 & -5 \end{pmatrix}$       E.  $\frac{1}{9} \begin{pmatrix} 1 & 2 \\ 2 & -5 \end{pmatrix}$

**Solusi: [E]**

$$AQ = 2B + A$$

$$\begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix} Q = 2 \begin{pmatrix} 1 & 2 \\ 4 & -1 \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 9 & 0 \end{pmatrix}$$

$$Q = \frac{1}{3} \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 3 & 3 \\ 9 & 0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 15 & 6 \\ 6 & -3 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 2 & -1 \end{pmatrix}$$

$$Q^{-1} = -\frac{1}{9} \begin{pmatrix} -1 & -2 \\ -2 & 5 \end{pmatrix} = \frac{1}{9} \begin{pmatrix} 1 & 2 \\ 2 & -5 \end{pmatrix}$$

9. Jika matriks  $A$  memenuhi persamaan  $A \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 0 & -6 \end{pmatrix}$ , maka determinan matriks  $A$  adalah ....

A. 17                      B. 12                      C. -10                      D. -12                      E. -17

**Solusi: [E]**

$$A \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 0 & -6 \end{pmatrix}$$

$$A = \begin{pmatrix} 4 & -2 \\ 0 & -6 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 4 & -2 \\ -5 & 3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 26 & -14 \\ 30 & -18 \end{pmatrix} = \begin{pmatrix} 13 & -7 \\ 15 & -9 \end{pmatrix}$$

$$|A| = -117 + 105 = -12$$

10. Jika matriks  $A$  memenuhi persamaan  $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 4 \\ -3 & 7 \end{pmatrix}$ , maka nilai  $bc$  adalah ....

A. -288                      B. -286                      C. -268                      D. 186                      E. 88

**Solusi: [B]**

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 4 \\ -3 & 7 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -2 & 4 \\ -3 & 7 \end{pmatrix} \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} -8 & 22 \\ -13 & 36 \end{pmatrix}$$

$$ad = -13 \times 22 = -286$$

11. Nilai dari  $\begin{vmatrix} -9 & -3 & 4 \\ -12 & 4 & 2 \\ -3 & -2 & -3 \end{vmatrix}$  adalah ....

A. -53                      B. -5                      C. 189                      D. 321                      E. 342

**Solusi: [E]**

$$\begin{vmatrix} -9 & -3 & 4 \\ -12 & 4 & 2 \\ -3 & -2 & -3 \end{vmatrix} = 108 + 18 + 96 - (-48 + 36 - 108) = 342$$

12. Diberikan matriks-matriks  $M = \begin{pmatrix} 1 & -2 \\ 5 & 0 \end{pmatrix}$  dan  $N = \begin{pmatrix} 2 & 4 \\ -1 & 5 \end{pmatrix}$ . Jika didefinisikan bahwa

$$f(M, N) = M^2 - N^2, \text{ maka } f(M - N, M + N) = \dots$$

A.  $\begin{pmatrix} -52 & 20 \\ 64 & -44 \end{pmatrix}$       B.  $\begin{pmatrix} -52 & -20 \\ 64 & -44 \end{pmatrix}$       C.  $\begin{pmatrix} -52 & -20 \\ -64 & 44 \end{pmatrix}$       D.  $\begin{pmatrix} -52 & -64 \\ 20 & -44 \end{pmatrix}$       E.  $\begin{pmatrix} -52 & 20 \\ -64 & -44 \end{pmatrix}$

**Solusi: [E]**

$$M + N = \begin{pmatrix} 1 & -2 \\ 5 & 0 \end{pmatrix} + \begin{pmatrix} 2 & 4 \\ -1 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 4 & 5 \end{pmatrix}$$

$$(M+N)^2 = \begin{pmatrix} 3 & 2 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 17 & 16 \\ 32 & 33 \end{pmatrix}$$

$$M-N = \begin{pmatrix} 1 & -2 \\ 5 & 0 \end{pmatrix} - \begin{pmatrix} 2 & 4 \\ -1 & 5 \end{pmatrix} = \begin{pmatrix} -1 & -6 \\ 6 & -5 \end{pmatrix}$$

$$(M-N)^2 = \begin{pmatrix} -1 & -6 \\ 6 & -5 \end{pmatrix} \begin{pmatrix} -1 & -6 \\ 6 & -5 \end{pmatrix} = \begin{pmatrix} -35 & 36 \\ -32 & -11 \end{pmatrix}$$

$$f(M-N, M+N) = (M-N)^2 - (M+N)^2 = \begin{pmatrix} -35 & 36 \\ -32 & -11 \end{pmatrix} - \begin{pmatrix} 17 & 16 \\ 32 & 33 \end{pmatrix} = \begin{pmatrix} -52 & 20 \\ -64 & -44 \end{pmatrix}$$

13. Jika  $I$  adalah matriks identitas ordo 2 yang memenuhi persamaan  $\begin{pmatrix} -11 & 6 \\ -6 & 4 \end{pmatrix} = p \begin{pmatrix} -3 & 2 \\ -2 & 2 \end{pmatrix} - qI$ , nilai

$$2p+q = \dots$$

- A. 8                      B. 7                      C. 6                      D. 5                      E. 4

**Solusi: [D]**

$$\begin{pmatrix} -11 & 6 \\ -6 & 4 \end{pmatrix} = p \begin{pmatrix} -3 & 2 \\ -2 & 2 \end{pmatrix} - qI$$

$$\begin{pmatrix} -11 & 6 \\ -6 & 4 \end{pmatrix} = p \begin{pmatrix} -3 & 2 \\ -2 & 2 \end{pmatrix} - q \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -3p-q & 2p \\ -2p & 2p-q \end{pmatrix}$$

$$2p = 6 \Leftrightarrow p = 3$$

$$2p - q = 4$$

$$2 \cdot 3 - q = 4$$

$$q = 2$$

$$2p + q = 2 \cdot 3 + 2 = 8$$

14. Diberikan matriks  $B = \begin{pmatrix} -4 & -3 \\ 2 & 1 \end{pmatrix}$  dan  $(B^{-1}A)^{-1} = \begin{pmatrix} 2 & 4 \\ 6 & -10 \end{pmatrix}$ , maka determinan matriks  $A^{-1}$  adalah...

- A. 152                      B. 87                      C. 65                      D. 22                      E. 2

**Solusi: [D]**

$$(B^{-1}A)^{-1} = \begin{pmatrix} 2 & 4 \\ 6 & -10 \end{pmatrix}$$

$$A^{-1}B = \begin{pmatrix} 2 & 4 \\ 6 & -10 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 2 & 4 \\ 6 & -10 \end{pmatrix} B^{-1} = \begin{pmatrix} 2 & 4 \\ 6 & -10 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & 3 \\ -2 & -4 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -6 & -10 \\ 26 & 58 \end{pmatrix} = \begin{pmatrix} -3 & -5 \\ 13 & 29 \end{pmatrix}$$

$$|A^{-1}| = -87 + 65 = -22$$

15. Jika matriks  $A = \begin{pmatrix} 2 & x & -1 \\ x & 2 & 1 \\ -3 & -1 & 1 \end{pmatrix}$ , dengan  $x < 0$  adalah matriks singular, maka nilai  $x$  adalah ....

- A. -8                      B. -4                      C. -2                      D. -1                      E. 0

**Solusi: [C]**

$$|A| = 4 - 3x + x - (6 - 2 + x^2) = 0$$

$$4 - 2x - 4 - x^2 = 0$$

$$x^2 + 2x = 0$$

$$x = 0 \text{ atau } x = -2$$

Karena  $x < 0$ , maka  $x = -2$

## B. URAIAN

16. Nabila, Sekar, dan Frana membeli apel dan jeruk di toko buah "RENDY". Nabila membeli 3 kg apel dan 5 kg jeruk seharga Rp300.000,00 dan Sekar membeli 2 kg apel dan 1 kg jeruk seharga Rp130.000,00. Jika Frana membeli 1 kg apel dan 2 kg jeruk dan ia membayar dengan 3 lembar uang lima puluh ribuan, berapakah uang kembaliannya? (Gunakan Metode Matriks)

**Solusi:**

Misalnya harga 1 kg apel dan jeruk masing-masing  $x$  dan  $y$  rupiah.

$$3x + 5y = 300.000$$

$$2x + y = 130.000$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3-10} \begin{pmatrix} 1 & -5 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 300.000 \\ 130.000 \end{pmatrix} = -\frac{1}{7} \begin{pmatrix} -350.000 \\ -210.000 \end{pmatrix} = \begin{pmatrix} 50.000 \\ 30.000 \end{pmatrix}$$

Jadi, uang kembaliannya Rp150.000,00 – (Rp50.000,00 + 2 × Rp30.000,00) = Rp40.000,00

17. Titik-titik (1,5), (-2,11), dan (2,19) terletak pada parabola  $y = ax^2 + bx + c$ . Tentukan nilai  $abc$  (Gunakan Kaidah Cramer).

**Solusi:**

$$(1,5) \rightarrow 5 = a + b + c$$

$$(-2,11) \rightarrow 11 = 4a - 2b + c$$

$$(2,1) \rightarrow 19 = 4a + 2b + c$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 4 & -2 & 1 \\ 4 & 2 & 1 \end{vmatrix} = -2 + 4 + 8 - (-8 + 2 + 4) = 10 + 2 = 12$$

$$\Delta a = \begin{vmatrix} 5 & 1 & 1 \\ 11 & -2 & 1 \\ 19 & 2 & 1 \end{vmatrix} = -10 + 19 + 22 - (-38 + 10 + 11) = 31 + 17 = 48$$

$$\Delta b = \begin{vmatrix} 1 & 5 & 1 \\ 4 & 11 & 1 \\ 4 & 19 & 1 \end{vmatrix} = 11 + 20 + 76 - (44 + 19 + 20) = 107 - 83 = 24$$

$$\Delta c = \begin{vmatrix} 1 & 1 & 5 \\ 4 & -2 & 11 \\ 4 & 2 & 19 \end{vmatrix} = -38 + 44 + 40 - (-40 + 22 + 76) = 46 - 58 = -12$$

$$a = \frac{\Delta a}{\Delta} = \frac{48}{12} = 4$$

$$b = \frac{\Delta b}{\Delta} = \frac{24}{12} = 2$$

$$c = \frac{\Delta c}{\Delta} = \frac{-12}{12} = -1$$

$$abc = 4 \cdot 2 \cdot (-1) = -8$$