

Mengenang Jejak Sebagian Kecil Bangsa Indonesia Yang Pernah Mengikuti Ujian Sekolah Pada Masa Silam
UJIAN PENGHABISAN SEKOLAH MENENGAH TINGKAT ATAS
TAHUN 1927

ALJABAR

1. HBS (Hogere Burger School) NI, 1927

Carilah x dan y dari

I. $({}^7\log x - {}^7\log y) \times {}^2\log 7 + 2 = {}^7\log 12 \times {}^2\log 7$

II. $28(x - y)^{-2} = 1 + 3(x - y)^{-1}$

Solusi:

$$({}^7\log x - {}^7\log y) \times {}^2\log 7 + 2 = {}^7\log 12 \times {}^2\log 7$$

$$\left({}^7\log \frac{x}{y}\right) \times {}^2\log 7 + 2 = {}^7\log 12 \times {}^2\log 7$$

$${}^2\log 7 \times {}^7\log \frac{x}{y} + 2 = {}^2\log 7 \times {}^7\log 12$$

$${}^2\log \frac{x}{y} + 2 = {}^2\log 12$$

$${}^2\log \frac{x}{y} - {}^2\log 12 = -2$$

$${}^2\log \frac{x}{12y} = -2$$

$$\frac{x}{12y} = 2^{-2} = \frac{1}{4}$$

$$y = \frac{x}{3} \dots (1)$$

$$28(x - y)^{-2} = 1 + 3(x - y)^{-1}$$

$$28 = (x - y)^2 + 3(x - y)$$

$$(x - y)^2 + 3(x - y) - 28 = 0$$

$$[(x - y) + 7][(x - y) - 4] = 0$$

$$x - y + 7 = 0 \dots (2) \text{ atau } x - y - 4 = 0 \dots (3)$$

Dari persamaan (1) dan (2) diperoleh:

$$x - \frac{x}{3} + 7 = 0$$

$$\frac{2x}{3} = -7$$

$$x = -\frac{21}{2} \text{ (ditolak, karena numerus logaritma harus lebih dari 0)}$$

Dari persamaan (1) dan (3) diperoleh:

$$x - \frac{x}{3} - 4 = 0$$

$$\frac{2x}{3} = 4$$

$$x = 6$$

$$y = \frac{6}{3} = 2$$

2. **AMS (Algemeene Middelbare School) afd B, 1927**

Pecahkan dengan tiada memakai sebuah daftar logaritma:

$${}^{0,4}\log x + {}^{2,5}\log(x+1) = 2$$

$${}^{0,4}\log x + {}^{2,5}\log(x+1) = 2$$

$$\frac{\log x}{\log 0,4} + \frac{\log(x+1)}{\log 2,5} = 2$$

$$\frac{\log x}{\log \frac{2}{5}} + \frac{\log(x+1)}{\log \frac{5}{2}} = 2$$

$$\frac{\log x}{\log 2 - \log 5} + \frac{\log(x+1)}{\log 5 - \log 2} = 2$$

$$\frac{\log(x+1)}{\log 5 - \log 2} - \frac{\log x}{\log 5 - \log 2} = 2$$

$$\frac{\log(x+1) - \log x}{\log 5 - \log 2} = 2$$

$$\log \frac{x+1}{x} = 2 \log \frac{5}{2} = \log \frac{25}{4}$$

$$\frac{x+1}{x} = \frac{25}{4}$$

$$4x + 4 = 25x$$

$$21x = 4$$

$$x = \frac{4}{21}$$

3. **HBS (Hogere Burger School) Nederland, 1927**

Pada persamaan $x^2 - 2mx + 3(m^2 - 4) = 0$ diketahui bahwa akar-akarnya sejati (real dan berbeda) dan bahwa jumlah kuadrat akar-akarnya dikalikan dengan hasil perbanyakkan akar-akar sama dengan $-102 + 120\sqrt{2}$. Hitunglah m dan akar-akar persamaan ini.

Solusi:

$$(x_1^2 + x_2^2)x_1x_2 = -102 + 120\sqrt{2}$$

$$\left[(x_1 + x_2)^2 - 2x_1x_2 \right] x_1x_2 = -102 + 120\sqrt{2}$$

$$\left[(2m)^2 - 2 \cdot 3(m^2 - 4) \right] 3(m^2 - 4) = -102 + 120\sqrt{2}$$

$$(4m^2 - 6m^2 + 24)(m^2 - 4) = -34 + 40\sqrt{2}$$

$$(-2m^2 + 24)(m^2 - 4) = -34 + 40\sqrt{2}$$

$$(m^2 - 12)(m^2 - 4) = 17 - 20\sqrt{2}$$

$$m^4 - 16m^2 + 48 = 17 - 20\sqrt{2}$$

$$m^4 - 16m^2 + 31 + 20\sqrt{2} = 0$$

$$m^2 = \frac{16 \pm \sqrt{256 - 124 - 80\sqrt{2}}}{2} = \frac{16 \pm \sqrt{132 - 80\sqrt{2}}}{2} = \frac{16 \pm 2\sqrt{33 - 20\sqrt{2}}}{2} = 8 \pm \sqrt{33 - 2\sqrt{200}}$$

$$= 8 \pm (\sqrt{25} - \sqrt{8})$$

$$m_1^2 = 8 + 5 - 2\sqrt{2} = 13 - 2\sqrt{2} \approx 10,2 \text{ atau } m_2^2 = 8 - 5 + 2\sqrt{2} = 3 + 2\sqrt{2} \approx 5,8$$

Selanjutnya,

$$x^2 - 2mx + 3(m^2 - 4) = 0$$

$$D = (-2m)^2 - 4 \cdot 1 \cdot 3(m^2 - 4) = 4m^2 - 12m^2 + 48 = -8m^2 + 48 > 0 \Leftrightarrow m^2 < 6$$

Karena $m^2 < 6$, maka m_1^2 ditolak dan m_2^2 (diterima).

$$m^2 = 3 + 2\sqrt{2}$$

$$m = \pm\sqrt{3 + 2\sqrt{2}} = \pm(\sqrt{2} + 1)$$

$$m = \sqrt{2} + 1 \rightarrow x^2 - 2mx + 3(m^2 - 4) = 0$$

$$x^2 - (2\sqrt{2} + 2)x + 3(3 + 2\sqrt{2} - 4) = 0$$

$$x^2 - (2\sqrt{2} + 2)x + (-3 + 6\sqrt{2}) = 0$$

$$x = \frac{2\sqrt{2} + 2 \pm \sqrt{12 + 8\sqrt{2} + 12 - 24\sqrt{2}}}{2} = \frac{2\sqrt{2} + 2 \pm \sqrt{24 - 16\sqrt{2}}}{2}$$

$$= \frac{2\sqrt{2} + 2 \pm 2\sqrt{6 - 4\sqrt{2}}}{2} = \sqrt{2} + 1 \pm \sqrt{6 - 2\sqrt{8}} = \sqrt{2} + 1 \pm (\sqrt{4} - \sqrt{2})$$

$$x_1 = \sqrt{2} + 1 + 2 - \sqrt{2} = 3 \text{ atau } x_2 = \sqrt{2} + 1 - 2 + \sqrt{2} = -1 + 2\sqrt{2}$$

$$m = -\sqrt{2} - 1 \rightarrow x^2 - 2mx + 3(m^2 - 4) = 0$$

$$x^2 + (2\sqrt{2} + 2)x + 3(3 + 2\sqrt{2} - 4) = 0$$

$$x^2 + (2\sqrt{2} + 2)x + (-3 + 6\sqrt{2}) = 0$$

$$x = \frac{-2\sqrt{2} - 2 \pm \sqrt{12 + 8\sqrt{2} + 12 - 24\sqrt{2}}}{2} = \frac{-2\sqrt{2} - 2 \pm \sqrt{24 - 16\sqrt{2}}}{2}$$

$$= \frac{-2\sqrt{2} - 2 \pm 2\sqrt{6 - 4\sqrt{2}}}{2} = -\sqrt{2} - 1 \pm \sqrt{6 - 2\sqrt{8}} = -\sqrt{2} - 1 \pm (\sqrt{4} - \sqrt{2})$$

$$x_3 = -\sqrt{2} - 1 + 2 - \sqrt{2} = 1 - 2\sqrt{2} \text{ atau } x_4 = -\sqrt{2} - 1 - 2 + \sqrt{2} = -3$$