

BAB 7 LIMIT

LIMIT FUNGSI ALJABAR:

TIPE 1: Teorema L'Hospital (Menggunakan Diferensial/Turunan)

Jika $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$ adalah bentuk tak tentu, maka $\lim_{x \rightarrow c} \frac{f'(x)}{g'(x)} = \lim_{x \rightarrow c} \frac{f''(x)}{g''(x)}$ dan seterusnya sampai diperoleh bentuk tentu.

Contoh 1:

Nilai $\lim_{x \rightarrow 3} \frac{x^3 - 27}{4x^2 - 3x - 24} = \dots$

- A. $\frac{9}{7}$ B. $\frac{7}{9}$ C. $\frac{2}{21}$ D. $\frac{2}{9}$ E. $\frac{7}{18}$

Solusi 1: [A]

Menguraikan/Memfaktorkan/Faktorisasi

$$\lim_{x \rightarrow 3} \frac{x^3 - 27}{4x^2 - 3x - 24} = \lim_{x \rightarrow 3} \frac{(x-3)(x^2 + 3x + 9)}{(x-3)(4x+9)} = \lim_{x \rightarrow 3} \frac{x^2 + 3x + 9}{4x+9} = \frac{3^2 + 3 \cdot 3 + 9}{4 \cdot 3 + 9} = \frac{27}{21} = \frac{9}{7}$$

Solusi 2: Care (Teorema L'Hospital)

$$\lim_{x \rightarrow 3} \frac{x^3 - 27}{4x^2 - 3x - 24} = \lim_{x \rightarrow 3} \frac{3x^2}{8x - 3} = \frac{3 \cdot 3^2}{4 \cdot 3 + 9} = \frac{27}{21} = \frac{9}{7}$$

Contoh 2:

Nilai $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{3x-5}-2} = \dots$

- A. $\frac{4}{3}$ B. 1 C. $\frac{2}{3}$ D. $\frac{1}{3}$ E. 0

Solusi 1: [A]

Mengalikan dengan bentuk sekawan

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{x-3}{\sqrt{3x-5}-2} &= \lim_{x \rightarrow 3} \frac{x-3}{\sqrt{3x-5}-2} \times \frac{\sqrt{3x-5}+2}{\sqrt{3x-5}+2} = \lim_{x \rightarrow 3} \frac{(x-3)(\sqrt{3x-5}+2)}{3x-5-4} = \lim_{x \rightarrow 3} \frac{(x-3)(\sqrt{3x-5}+2)}{3(x-3)} \\ &= \lim_{x \rightarrow 3} \frac{\sqrt{3x-5}+2}{3} = \frac{\sqrt{3 \cdot 3-5}+2}{3} = \frac{2+2}{3} = \frac{4}{3} \end{aligned}$$

Solusi 2: Care (Teorema L'Hospital)

$$\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{3x-5}-2} = \lim_{x \rightarrow 3} \frac{1}{\frac{3}{2\sqrt{3x-5}}} = \lim_{x \rightarrow 3} \frac{2\sqrt{3x-5}}{3} = \frac{2\sqrt{3 \cdot 3-5}}{3} = \frac{2+2}{3} = \frac{4}{3}$$

Contoh 3:

Nilai $\lim_{x \rightarrow 2} \frac{\sqrt{4x+9} - \sqrt{x^2-3}}{x+2} = \dots$

- A. 8 B. 7 C. 6 D. 4 E. 2

Solusi 1: [D]

Mengalikan dengan bentuk sekawan

$$\begin{aligned} \lim_{x \rightarrow -2} \frac{\sqrt{4x+9} - \sqrt{x^2-3}}{x+2} &= \lim_{x \rightarrow -2} \frac{\sqrt{4x+9} - \sqrt{x^2-3}}{x+2} \times \frac{\sqrt{4x+9} + \sqrt{x^2-3}}{\sqrt{4x+9} + \sqrt{x^2-3}} \\ &= \lim_{x \rightarrow -2} \frac{4x+9-x^2+3}{(x+2)(\sqrt{4x+9} + \sqrt{x^2-3})} \\ &= \lim_{x \rightarrow -2} \frac{(6-x)(x+2)}{(x+2)(\sqrt{4x+9} + \sqrt{x^2-3})} = \lim_{x \rightarrow -2} \frac{6-x}{\sqrt{4x+9} + \sqrt{x^2-3}} = \frac{6+2}{\sqrt{4(-2)+9} + \sqrt{(-2)^2-3}} = \frac{8}{2} = 4 \end{aligned}$$

Solusi 2: Care (Teorema L'Hospital)

$$\lim_{x \rightarrow -2} \frac{\sqrt{4x+9} - \sqrt{x^2-3}}{x+2} = \lim_{x \rightarrow -2} \frac{\frac{4}{2\sqrt{4x+9}} - \frac{2x}{2\sqrt{x^2-3}}}{1} = \frac{2}{\sqrt{4(-2)+9}} - \frac{-2}{\sqrt{(-2)^2-3}} = 2+2=4$$

TIPE 2: Dengan Membagi Pangkat Tertinggi

$$\lim_{x \rightarrow \infty} \frac{ax^m + a_1x^{m-1} + a_2x^{m-2} + \dots}{bx^n + b_1x^{n-1} + b_2x^{n-2} + \dots} = \begin{cases} \frac{a}{b}, & \text{jika } m = n \\ \infty, & \text{jika } m > n \\ 0, & \text{jika } m < n \end{cases}$$

Contoh 1:

Nilai dari $\lim_{x \rightarrow \infty} \frac{(3x-2)^3}{(2x^2+x-1)(3x+4)} = \dots$

- A. $\frac{1}{2}$ B. $\frac{3}{2}$ C. $\frac{9}{2}$ D. $+\infty$ E. $-\infty$

Solusi 1: [C]

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{(3x-2)^3}{(2x^2+x-1)(3x+4)} \left(\text{Bentuk } \frac{\infty}{\infty} \right) &= \lim_{x \rightarrow \infty} \frac{(3x-2)^3}{(2x^2+x-1)(3x+4)} = \lim_{x \rightarrow \infty} \frac{27x^3 - 54x^2 + 36x - 8}{6x^3 + 11x^2 + x - 4} \\ &= \lim_{x \rightarrow \infty} \frac{27 - \frac{54}{x} + \frac{36}{x^2} - \frac{8}{x^3}}{6 + \frac{11}{x} + \frac{1}{x^2} - \frac{4}{x^3}} = \lim_{x \rightarrow \infty} \frac{27 - \frac{54}{\infty} + \frac{36}{\infty^2} - \frac{8}{\infty^3}}{6 + \frac{11}{\infty} + \frac{1}{\infty^2} - \frac{4}{\infty^3}} = \frac{27}{6} = \frac{9}{2} \end{aligned}$$

Solusi 2: Care

$$\lim_{x \rightarrow \infty} \frac{(4x^2-5x)(x^2+3)}{2x^3+x^2-1} \left(\text{Bentuk } \frac{\infty}{\infty} \right) = \frac{3^3}{2 \times 3} = \frac{9}{2} \quad (\text{Pangkat pembilang dan penyebut sama})$$

Contoh 2:

Nilai $\lim_{x \rightarrow \infty} \left(\frac{16x^3 - x^2 - 5x + 8}{3 + 4x^2 - 4x^3} \right)^5 = \dots$

- A. -1.024 B. -512 C. 512 D. 624 E. 1.024

Solusi 1: [A]

Membagi dengan pangkat tertinggi

$$\lim_{x \rightarrow \infty} \left(\frac{16x^3 - x^2 - 5x + 8}{3 + 4x^2 - 4x^3} \right)^5 \left(\text{Bentuk } \frac{\infty}{\infty} \right) = \left(\lim_{x \rightarrow \infty} \frac{16 - \frac{1}{x} - \frac{5}{x^2} + \frac{8}{x^3}}{\frac{3}{x^3} + \frac{4}{x} - 4} \right)^5 = \left(\lim_{x \rightarrow \infty} \frac{16 - \frac{1}{\infty} - \frac{5}{\infty^2} + \frac{8}{\infty^3}}{\frac{3}{\infty^3} + \frac{4}{\infty} - 4} \right)^5$$

$$= \left(\frac{16 - 0 - 0 + 0}{0 + 0 - 4} \right)^5 = \left(\frac{16}{-4} \right)^5 = (-4)^5 = -1.024$$

Solusi 2: Care

$$\lim_{x \rightarrow \infty} \left(\frac{16x^3 - x^2 - 5x + 8}{3 + 4x^2 - 4x^3} \right)^5 \left(\text{Bentuk } \frac{\infty}{\infty} \right) = \left(\frac{16}{-4} \right)^5 = (-4)^5 = -1.024 \text{ (Pangkat pembilang dan penyebut sama)}$$

Contoh 3:

Nilai dari $\lim_{x \rightarrow \infty} \frac{(4x^2 - 5x)(x^2 + 3)}{2x^3 + x^2 - 1} = \dots$

- A. 1 B. 2 C. 4 D. ∞ E. $-\infty$

Solusi 1: [D]

$$\lim_{x \rightarrow \infty} \frac{(4x^2 - 5x)(x^2 + 3)}{2x^3 + x^2 - 1} \left(\text{Bentuk } \frac{\infty}{\infty} \right) = \lim_{x \rightarrow \infty} \frac{4x^4 - 5x^3 + 12x^2 - 15x}{2x^3 + x^2 - 1} = \lim_{x \rightarrow \infty} \frac{27 - \frac{5}{x} + \frac{12}{x^2} - \frac{15}{x^3}}{\frac{2}{x} + \frac{1}{x^2} - \frac{1}{x^4}}$$

$$= \lim_{x \rightarrow \infty} \frac{27}{0} = \infty$$

Solusi 2: Care

$$\lim_{x \rightarrow \infty} \frac{(4x^2 - 5x)(x^2 + 3)}{2x^3 + x^2 - 1} \left(\text{Bentuk } \frac{\infty}{\infty} \right) = \lim_{x \rightarrow \infty} \frac{4x^4 - 5x^3 + 12x^2 - 15x}{2x^3 + x^2 - 1} = \infty \text{ (Pangkat pembilang > pangkat penyebut)}$$

Contoh 4:

Nilai dari $\lim_{x \rightarrow \infty} \frac{8x^2 - 5x + 9}{4x^3 + 2x^2 - 1} = \dots$

- A. 0 B. 2 C. 4 D. ∞ E. $-\infty$

Solusi 1: [A]

$$\lim_{x \rightarrow \infty} \frac{8x^2 - 5x + 9}{4x^3 + 2x^2 - 1} \left(\text{Bentuk } \frac{\infty}{\infty} \right) = \lim_{x \rightarrow \infty} \frac{\frac{8}{x} - \frac{5}{x^2} + \frac{9}{x^3}}{4 + \frac{2}{x} - \frac{1}{x^3}} = \lim_{x \rightarrow \infty} \frac{0}{4} = 0$$

Solusi 2: Care

$$\lim_{x \rightarrow \infty} \frac{8x^2 - 5x + 9}{4x^3 + 2x^2 - 1} \left(\text{Bentuk } \frac{\infty}{\infty} \right) = 0 \text{ (Pangkat pembilang < pangkat penyebut)}$$

TIPE 3: $\lim_{x \rightarrow \infty} \left(\sqrt{ax^2 + bx + c} - \sqrt{ax^2 + px + q} \right)$

Contoh 1:

Nilai $\lim_{x \rightarrow \infty} \left(\sqrt{4x^2 - 6x + 9} - \sqrt{4x^2 + 9x} \right) = \dots$

- A. $-\frac{15}{4}$ B. $-\frac{4}{15}$ C. 0 D. $\frac{4}{5}$ E. ∞

Solusi 1: [A]

Mengalikan dengan bentuk sekawan

$$\begin{aligned} \lim_{x \rightarrow \infty} (\sqrt{4x^2 - 6x + 9} - \sqrt{4x^2 + 9x}) &= \lim_{x \rightarrow \infty} (\sqrt{4x^2 - 6x + 9} - \sqrt{4x^2 + 9x}) \times \frac{\sqrt{4x^2 - 6x + 9} + \sqrt{4x^2 + 9x}}{\sqrt{4x^2 - 6x + 9} + \sqrt{4x^2 + 9x}} \\ &= \lim_{x \rightarrow \infty} \frac{4x^2 - 6x + 9 - (4x^2 + 9x)}{\sqrt{4x^2 - 6x + 9} + \sqrt{4x^2 + 9x}} = \lim_{x \rightarrow \infty} \frac{4x^2 - 6x + 9 - 4x^2 - 9x}{\sqrt{4x^2 - 6x + 9} + \sqrt{4x^2 + 9x}} \\ &= \lim_{x \rightarrow \infty} \frac{-15x + 9}{\sqrt{4x^2 - 6x + 9} + \sqrt{4x^2 + 9x}} = \lim_{x \rightarrow \infty} \frac{-15 + \frac{9}{x}}{\sqrt{4 - \frac{6}{x} + \frac{9}{x^2}} + \sqrt{4 + \frac{9}{x}}} \\ &= \lim_{x \rightarrow \infty} \frac{-15 + \frac{9}{\infty}}{\sqrt{4 - \frac{6}{\infty} + \frac{9}{\infty^2}} + \sqrt{4 + \frac{9}{\infty}}} = \frac{-15 + 0}{\sqrt{4 - 0 + 0} + \sqrt{4 + 0}} = \frac{-15}{2 + 2} = -\frac{15}{4} \end{aligned}$$

Solusi 2: Care

$$\lim_{x \rightarrow \infty} (\sqrt{4x^2 - 6x + 9} - \sqrt{4x^2 + 9x}) = \lim_{x \rightarrow \infty} \left[\sqrt{\left(2x - \frac{3}{2}\right)^2} - \sqrt{\left(2x + \frac{9}{4}\right)^2} \right] = \lim_{x \rightarrow \infty} \left(2x - \frac{3}{2} - 2x - \frac{9}{4} \right) = -\frac{15}{4}$$

Contoh 2:

Nilai $\lim_{x \rightarrow \infty} (3x + 5 - \sqrt{9x^2 + 18x - 5}) = \dots$

- A. $\frac{1}{2}$ B. 1 C. 2 D. 3 E. ∞

Solusi: [C]

Care:

$$\lim_{x \rightarrow \infty} (3x + 5 - \sqrt{9x^2 + 18x - 5}) = \lim_{x \rightarrow \infty} [3x + 5 - \sqrt{(3x + 3)^2}] = \lim_{x \rightarrow \infty} (3x + 5 - 3x - 3) = 2$$

Contoh 3:

Nilai dari $\lim_{x \rightarrow \infty} (\sqrt{4x^2 + 8x} - \sqrt{x^2 + 1} - \sqrt{x^2 + x}) = \dots$

- A. $\frac{5}{2}$ B. 2 C. $\frac{3}{2}$ D. 1 E. $\frac{1}{2}$

Solusi: [C]

Care:

$$\lim_{x \rightarrow \infty} (\sqrt{4x^2 + 8x} - \sqrt{x^2 + 1} - \sqrt{x^2 + x}) = \lim_{x \rightarrow \infty} \left(2x + 2 - x - x - \frac{1}{2} \right) = \frac{3}{2}$$

Contoh 4:

Nilai $\lim_{x \rightarrow \infty} (\sqrt{25x^2 - 6x + 2013} - \sqrt{4x^2 + 4} + \sqrt{x^2 + 2x} - \sqrt{16x^2 - 6x + 2013}) = \dots$

- A. 0 B. 1 C. 2 D. 3 E. ∞

Solusi: [B]

Care:

$$\lim_{x \rightarrow \infty} \left(\sqrt{25x^2 - 30x + 2013} - \sqrt{4x^2 + 4} + \sqrt{x^2 + 2x} - \sqrt{16x^2 - 24x + 2013} \right)$$

$$= \lim_{x \rightarrow \infty} (5x - 3 - 2x + x + 1 - 4x + 3) = 1$$

LIMIT FUNGSI TRIGONOMETRI:

Rumus Aproksimasi

$$1. \lim_{x \rightarrow c} \sin ax \rightarrow \lim_{x \rightarrow c} ax \qquad 2. \lim_{x \rightarrow c} \tan ax \rightarrow \lim_{x \rightarrow c} ax \qquad 3. \lim_{x \rightarrow c} (1 - \cos ax) \rightarrow \lim_{x \rightarrow c} \frac{1}{2} (ax)^2$$

Contoh 1:

Nilai dari $\lim_{x \rightarrow 0} \frac{\sin 4x}{2x} = \dots$

- A. 0 B. 2 C. 4 D. ∞ E. $-\infty$

Solusi 1:[B]

Faktorisasi

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{2x} \left(\text{Bentuk } \frac{0}{0} \right) = 2 \lim_{x \rightarrow 0} \frac{\sin 4x}{4x} = 2 \cdot 1 = 2$$

Solusi 2: Teorema L'Hospital

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{2x} \left(\text{Bentuk } \frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{4 \cos 4x}{2} = \frac{4 \cos 0}{2} = 2$$

Solusi 3: Care

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{2x} \left(\text{Bentuk } \frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{4x}{2x} = 2$$

Contoh 2:

Nilai dari $\lim_{x \rightarrow 0} \frac{\sin 4x \tan 6x}{2x \sin 3x} = \dots$

- A. 0 B. 2 C. 4 D. ∞ E. $-\infty$

Solusi 1: [C]

Faktorisasi

$$\lim_{x \rightarrow 0} \frac{\sin 4x \tan 6x}{2x \sin 3x} \left(\text{Bentuk } \frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{2 \sin 4x}{4x} \times \lim_{x \rightarrow 0} \frac{6 \tan 6x}{6x} \times \lim_{x \rightarrow 0} \frac{3x}{3 \sin 3x} = \frac{2 \times 6}{3} = 4$$

Solusi 2: Care

$$\lim_{x \rightarrow 0} \frac{\sin 4x \tan 6x}{2x \sin 3x} \left(\text{Bentuk } \frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{4x \cdot 6x}{2x \cdot 3x} = 4$$

Contoh 3:

Nilai dari $\lim_{x \rightarrow 3} \frac{(x^2 - 9) \sin(3x - 9)}{(x^2 - 2x - 3)} = \dots$

- A. $\frac{3}{8}$ B. $\frac{9}{8}$ C. $\frac{9}{16}$ D. ∞ E. $-\infty$

Solusi 1: [B]

Faktorisasi

$$\lim_{x \rightarrow 3} \frac{(x^2 - 9) \sin(3x - 9)}{(x^2 - 2x - 3)^2} \left(\text{Bentuk } \frac{0}{0} \right) = \lim_{x \rightarrow 3} \frac{(x - 3)(x + 3) \sin 3(x - 3)}{(x - 3)^2 (x + 1)^2} = \lim_{x \rightarrow 3} \frac{(x + 3)}{(x + 1)^2} \times \lim_{x \rightarrow 3} \frac{3 \sin 3(x - 3)}{3(x - 3)}$$

$$= \frac{3+3}{(3+1)^2} \times 3 \times 1 = \frac{9}{8}$$

Solusi 2: Care

$$\lim_{x \rightarrow 3} \frac{(x^2 - 9)\sin(3x - 9)}{(x^2 - 2x - 3)^2} \left(\text{Bentuk } \frac{0}{0} \right) = \lim_{x \rightarrow 3} \frac{(x-3)(x+3)(3x-9)}{(x-3)^2(x+1)^2} = \lim_{x \rightarrow 3} \frac{3(x+3)}{(x+1)^2} = \frac{3(3+3)}{(3+1)^2} = \frac{9}{8}$$

Contoh 4:

Nilai dari $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{\sin 3x \tan 2x} = \dots$

- A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{2}{3}$ D. ∞ E. $-\infty$

Solusi 1: [C]

Faktorisasi

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{1 - \cos 4x}{\sin 3x \tan 2x} \left(\text{Bentuk } \frac{0}{0} \right) &= \lim_{x \rightarrow 0} \frac{2 \sin^2 2x}{\sin 3x \tan 2x} = \frac{4}{3} \times \lim_{x \rightarrow 0} \frac{3x}{\sin 3x} \times \lim_{x \rightarrow 0} \frac{2x}{\tan 2x} \times \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \times \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \\ &= \frac{4}{3} \times 1 \times 1 \times 1 \times 1 = \frac{4}{3} \end{aligned}$$

Solusi 2: Care

$$\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{\sin 3x \tan 2x} \left(\text{Bentuk } \frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{\frac{1}{2}(4x)^2}{(3x)(2x)} = \frac{4}{3}$$

Contoh 5:

Nilai dari $\lim_{x \rightarrow 0} \frac{\sin 2x + \tan 4x}{5x + \tan 3x} = \dots$

- A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. $\frac{3}{4}$ D. ∞ E. $-\infty$

Solusi 1: [C]

Faktorisasi

$$\lim_{x \rightarrow 0} \frac{\sin 2x + \tan 4x}{5x + \tan 3x} \left(\text{Bentuk } \frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{\frac{\sin 2x}{x} + \frac{\tan 4x}{x}}{5 + \frac{\tan 3x}{x}} = \frac{\lim_{x \rightarrow 0} \frac{2 \sin 2x}{2x} + \lim_{x \rightarrow 0} \frac{4 \tan 4x}{4x}}{\lim_{x \rightarrow 0} 5 + \lim_{x \rightarrow 0} \frac{3 \tan 3x}{3x}} = \frac{2 + 4}{5 + 3} = \frac{3}{4}$$

Solusi 2: Care

$$\lim_{x \rightarrow 0} \frac{\sin 2x + \tan 4x}{5x + \tan 3x} \left(\text{Bentuk } \frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{2x + 4x}{5x + 3x} = \frac{3}{4}$$

Contoh 6:

Nilai dari $\lim_{x \rightarrow 0} \frac{12x^3}{\tan 2x - \sin 2x} = \dots$

- A. $\frac{3}{4}$ B. $\frac{5}{4}$ C. $\frac{7}{4}$ D. 3 E. 4

Solusi 1: [D]

Faktorisasi

$$\lim_{x \rightarrow 0} \frac{12x^3}{\tan 2x - \sin 2x} \left(\text{Bentuk } \frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{12x^3}{\tan 2x(1 - \cos 2x)} = \lim_{x \rightarrow 0} \frac{12x^3}{\tan 2x \cdot 2 \sin^2 x} = \lim_{x \rightarrow 0} \frac{6x^3}{\tan 2x \sin^2 x}$$

$$= 3 \cdot \lim_{x \rightarrow 0} \frac{2x}{\tan 2x} \cdot \lim_{x \rightarrow 0} \frac{x}{\sin x} \cdot \lim_{x \rightarrow 0} \frac{x}{\sin x} = 3 \cdot 1 \cdot 1 \cdot 1 = 3$$

Solusi 2: Care

$$\lim_{x \rightarrow 0} \frac{12x^3}{\tan 2x - \sin 2x} \left(\text{Bentuk } \frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{12x^3}{\tan 2x(1 - \cos 2x)} = \lim_{x \rightarrow 0} \frac{12x^3}{2x \cdot \frac{1}{2}(2x)^2} = 3$$

LATIHAN SOAL-SOAL

1. **UN 2013**

Nilai $\lim_{x \rightarrow \infty} \left[\sqrt{9x^2 - 6x - 1} - (3x + 1) \right] = \dots$

- A. -2 B. -1 C. 0 D. 1 E. 2

2. **UN 2013**

Nilai $\lim_{x \rightarrow 0} \frac{4 \sin^2 2x}{x \tan 2x} = \dots$

- A. -8 B. -4 C. 0 D. 4 E. 8

3. **UN 2013**

Nilai $\lim_{x \rightarrow 0} \frac{1 - \cos^2 4x}{2x \tan 2x} = \dots$

- A. 2 B. 4 C. 6 D. 10 E. 14

4. **UN 2013**

Nilai dari $\lim_{x \rightarrow \infty} \left(\sqrt{4x^2 - 8x + 6} - \sqrt{4x^2 + 16x - 3} \right) = \dots$

- A. -6 B. -3 C. 4 D. 6 E. 10

5. **UN 2013**

Nilai dari $\lim_{x \rightarrow -2} \frac{(x^2 - 4) \cdot \tan(x + 2)}{\sin^2(x + 2)} = \dots$

- A. -4 B. -3 C. 0 D. 4 E. ∞

6. **UN 2013**

Nilai $\lim_{x \rightarrow 3} \frac{x \tan(2x - 6)}{\sin(x - 3)} = \dots$

- A. 0 B. $\frac{1}{2}$ C. 2 D. 3 E. 6

7. **UN 2013**

Nilai dari $\lim_{x \rightarrow \infty} \left[(2x - 1) - \sqrt{4x^2 - 6x - 5} \right] = \dots$

- A. 4 B. 2 C. 1 D. $\frac{1}{2}$ E. $\frac{1}{4}$

8. **UN 2013**

Nilai $\lim_{x \rightarrow 1} \frac{\sin^2(x - 1)}{x^2 - 2x + 1} = \dots$

- A. 0 B. 1 C. 2 D. 4 E. ∞

9. UN 2013

Nilai dari $\lim_{x \rightarrow \infty} [\sqrt{4x^2 + 4x - 3} - (2x - 5)] = \dots$

- A. -6 B. -4 C. -1 D. 4 E. 6

10. UN 2013

Nilai dari $\lim_{x \rightarrow \infty} (\sqrt{25x^2 - 9x - 16} - 5x + 3) = \dots$

- A. $-\frac{39}{10}$ B. $-\frac{9}{10}$ C. $\frac{21}{10}$ D. $\frac{39}{10}$ E. ∞

11. UN 2013

Nilai dari $\lim_{x \rightarrow 2} \frac{(2x+1)\tan(x-2)}{x^2-4} = \dots$

- A. 5 B. 2,5 C. 2 D. 1,5 E. 1,25

12. UN 2013

Nilai $\lim_{x \rightarrow \infty} (\sqrt{4x^2 + 3x + 4} - 2x + 1) = \dots$

- A. $-\frac{7}{4}$ B. 0 C. $\frac{3}{4}$ D. $\frac{7}{4}$ E. ∞

13. UN 2013

Nilai $\lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{x \sin 2x} = \dots$

- A. 4 B. 2 C. 0 D. -2 E. -4

14. UN A35, D74, dan E81 2012

Nilai $\lim_{x \rightarrow 3} \frac{2 - \sqrt{x+1}}{x-3} = \dots$

- A. $-\frac{1}{4}$ B. $-\frac{1}{2}$ C. 1 D. 2 E. 4

15. UN A35 2012

Nilai $\lim_{x \rightarrow 0} \frac{x \tan x}{1 - \cos 2x} = \dots$

- A. $-\frac{1}{2}$ B. 0 C. $\frac{1}{2}$ D. 1 E. 2

16. UN B47 2012

Nilai $\lim_{x \rightarrow 0} \frac{5x}{3 - \sqrt{9+x}} = \dots$

- A. -30 B. -27 C. 15 D. 30 E. 36

17. UN B47 2012

Nilai $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x \tan 2x} = \dots$

- A. -2 B. -1 C. 0 D. 1 E. 2

18. UN C61, D74, dan E81 2012

Nilai $\lim_{x \rightarrow 0} \frac{\cos 4x - 1}{x \tan 2x} = \dots$

- A. 4 B. 2 C. -1 D. -2 E. -4

19. UN A P12 dan B45 2011

Nilai $\lim_{x \rightarrow 4} \frac{(x-4)}{\sqrt{x}-2} = \dots$

- A. 0 B. 4 C. 8 D. 12 E. 16

20. **UN A P12 dan B45 2011**

Nilai $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{2x \sin 2x} = \dots$

- A. $\frac{1}{8}$ B. $\frac{1}{6}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. 1

21. **UN A P 12 2010**

Nilai $\lim_{x \rightarrow 0} \left(\frac{4x}{\sqrt{1-2x} - \sqrt{1+2x}} \right) = \dots$

- A. -2 B. 0 C. 1 D. 2 E. 4

22. **UN A P 12 2010**

Nilai $\lim_{x \rightarrow 0} \left(\frac{\sin 4x - \sin 2x}{6x} \right) = \dots$

- A. 1 B. $\frac{2}{3}$ C. $\frac{1}{2}$ D. $\frac{1}{3}$ E. $\frac{1}{6}$

23. **UN B P45 2010**

Nilai $\lim_{x \rightarrow 0} \left(\frac{1 - \cos 2x}{x^2} \right) = \dots$

- A. 2 B. 1 C. $\frac{1}{2}$ D. $\frac{1}{4}$ E. -2

24. **UN AP 12 dan B 45 2009**

Nilai $\lim_{x \rightarrow 3} \frac{x^2 - 9}{\sqrt{10 + 2x} - (x + 1)} = \dots$

- A. -8 B. -6 C. 4 D. 6 E. 8

25. **UN AP dan B 45 12 2009**

Nilai $\lim_{x \rightarrow 1} \frac{(x^2 - 1) \sin 2(x-1)}{-2 \cdot \sin^2(x-1)} = \dots$

- A. -2 B. -1 C. $-\frac{1}{2}$ D. $-\frac{1}{4}$ E. 0

26. **UN A P 12 dan B 45 2008**

Nilai dari $\lim_{x \rightarrow 2} \frac{x^3 - 4x}{x - 2} = \dots$

- A. 32 B. 16 C. 8 D. 4 E. 2